

The Standard Model of Particle Physics



Steven Weinberg and his legacy

R. Barbieri
GGI, 19-01-2022

A one-hundred-years story, not over yet

- The making of the SM (circa 1910/20 $\rightarrow$ 1973(?))
- The amazing success of the SM, so far (1974 $\rightarrow$ now)
- The future of the SM (now $\rightarrow$)

The making of the SM (circa 1910/20 → 1973(?))

A broad view and Weinberg legacy

The content

- A. An effective theory of the weak (and the strong) interactions
- B. A full fledged theory of the weak and the strong interactions

The methodology

- 1. A phenomenological approach: new concepts to agree with exp.s
- 2. The search for mathematical/logical consistency

Weinberg a leading actor in B, but in A as well
and a master of equilibrium between 1 and 2

An effective theory of the weak and the strong int.s

1910/20 The continuous spectrum in β -decay (an energy crisis?)

1930 Pauli "neutron": a new particle of spin 1/2

1934 Fermi: Tentativo di una teoria della emissione di raggi β

$$\mathcal{H}_I = g[\tau_- \Psi_e^+(x) \Psi_\nu(x) + \tau_+ \Psi_\nu^+(x) \Psi_e(x)]$$

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$$\mathcal{H}_I = g[\tau_- \Psi_e^+(x) \Psi_\nu(x) + \tau_+ \Psi_\nu^+(x) \Psi_e(x)]$$

1936 $\mathcal{H}_I = g \Sigma_i (\bar{\Psi}_p(x) \mathcal{O}_i \Psi_n(x)) (\bar{\Psi}_e(x) \mathcal{O}_i \Psi_\nu(x)) \quad i = S, P, V, A, T$

soon recognised to be problematic in its h.e. behaviour

$$\frac{d\sigma}{d\Omega}(\nu p \rightarrow en) \propto G_F^2 p_\nu^2$$

and gradually reduced (1950/60) to the current-current form

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} J^\mu(x) J_\mu^+(x), \quad J^\mu(x) = l^\mu(x) + h^\mu(x)$$

$$l^\mu(x) = \bar{\nu}(x) \gamma_\mu (1 + \gamma_5) e(x) \quad \text{What about } h^\nu(x)?$$

What about $h^\nu(x)$?

$h^\mu = \mathcal{V}_\mu + \mathcal{A}_\mu$ = a current associated with a symmetry of the S.I.

1958 Conserved Vector Current $\mathcal{V}_\mu = I_\mu^+$
making possible to establish β -decay $\leftrightarrow$ μ -decay universality

1960/62 PCAC: chiral invariance spontaneously broken
 π 's as Nambu-Goldstone bosons?

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1963 $\beta \longleftrightarrow \mu$ -decay universality extended to strange particles

$$h_\mu = \cos\theta h_\mu^{(\Delta S=0)} + \sin\theta h_\mu^{(\Delta S=1)} \quad \text{Cabibbo}$$

1964 The algebra of currents (CA)

$$U(3) \times U(3) = U(1) \times U(1) \times SU(3) \times SU(3) \rightarrow U(1)_V \times SU(3)_V$$

1965/68 **Weinberg**: from CA to effective Chiral Lagrangians
applied to π, B, ρ . "CA without CA"

Towards a full fledged gauge theory

1920/50 From $U(1)_{em}$ (late 40's) to speculations on $SU(2)_I$

1954 Yang-Mills gauge theory for $SU(2)$

1957 Schwinger: from $\mathcal{H}_I = \frac{G_F}{\sqrt{2}} J^\mu(x) J_\mu^+(x)$ to $\mathcal{H}_I = g J^\mu(x) W_\mu^- + h.c.$
A triplet of vectors $Z_\mu^{\pm,0}, \quad Z_\mu^0 \stackrel{?}{=} A_\mu$

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1961 Glashow: $SU(2) \times U(1) \quad [\vec{O}, S] = 0 \quad A_\mu = \cos\theta' Z_\mu^S + \sin\theta' Z_\mu^3$
Explicit vector masses where from?

1961 Yang-Mills extended to an arbitrary Lie algebra

1964 Brout-Englert-Higgs mechanism
No massless particles anymore! A massive scalar

1967 Weinberg: A model of leptons

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu (\partial_\mu - ig'B_\mu)R - L\gamma^\mu (\partial_\mu + ig\vec{t} \cdot \vec{A}_\mu - i\frac{1}{2}g'B_\mu)L \\ & - \frac{1}{2}|\partial_\mu \varphi - ig\vec{A}_\mu \cdot \vec{t}\varphi + i\frac{1}{2}g'B_\mu\varphi|^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1^2\varphi^\dagger\varphi + h(\varphi^\dagger\varphi)^2. \quad (4) \end{aligned}$$

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$$- \frac{1}{2}|\partial_\mu \varphi - ig\vec{A}_\mu \cdot \vec{t} \varphi + i\frac{1}{2}g'B_\mu \varphi|^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1^2 \varphi^\dagger \varphi + h(\varphi^\dagger \varphi)^2. \quad (4)$$

Spontaneous symmetry breaking: $(SU(2) \times U(1) \rightarrow U(1)_{em})$

- used for the first time in weak interactions
- in generating vector boson masses

$$m_W > \frac{e^2}{4\sqrt{2}G_F} \approx 40 \text{ GeV} \quad (g = e, g' \rightarrow \infty)$$

$$m_Z > \frac{e^2}{2\sqrt{2}G_F} \approx 80 \text{ GeV} \quad (g = g' = \sqrt{2}e)$$

- in generating fermion masses

Is this model renormalisable? Yes, t'Hooft, Veltman 1971

How to include hadrons?

1970/73 The inclusion of hadrons (and the setting of QCD)

1970 $\Delta S = 1$ ($K_L \rightarrow \mu\mu$) and $\Delta S = 2$ ($K_0 - \bar{K}_0$) need a low cutoff $\Lambda \approx 2 \div 3 \text{ GeV}$

GIM:
$$h_\mu = \bar{u}\gamma_\mu(1 + \gamma_5)(c_\theta d + s_\theta s) + \bar{c}\gamma_\mu(1 + \gamma_5)(c_\theta s - s_\theta d)$$

1972/73 G,W;P Asymptotic Freedom

1972/73 G,F,B,L “Coloured” quarks coupled to an octet of “gluons”

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A) $G = G_S \times SU(2) \times U(1)$

B) Ψ 's form a non-chiral rep of G_S

C) ϕ 's are G_S -neutral, so that $\langle \phi \rangle$ leaves G_S unbroken

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Why WI do not produce P and S violations of order α ?

$$\mathcal{L}_{strong} = -\bar{\Psi} Z_\Psi \gamma_\mu D_\mu \Psi - \bar{\Psi} m \Psi - 1/4 Z_{ab}^A F_{a\mu\nu} F^{b\mu\nu}$$

By Ψ redefinition, P and S are accidental symmetries of the strong (and the em) interactions

All of Particle Physics in 1 page

1. Symmetry group $L \times \mathcal{G}$

L = Lorentz (space-time)

$\mathcal{G} = SU(3) \times SU(2) \times U(1)$ (local)

2. Particle content (rep.s of $L \times \mathcal{G}$)

	h	Q	L	u	d	e
Lorentz	0	$1/2_L$	$1/2_L$	$1/2_R$	$1/2_R$	$1/2_R$
$SU(3)$	1	3	1	3	3	1
$SU(2)$	2	2	2	1	1	1
$U(1)$	$-1/2$	$1/6$	$-1/2$	$2/3$	$-1/3$	-1

3. All “operators” (local products of $\Phi, \partial_\mu \Phi$) in $\mathcal{L}$ of dimension ≤ 4

$$\hbar = c = 1 \Rightarrow [A_\mu] = [\phi] = [\partial_\mu] = M, \quad [\Psi] = M^{3/2}, \quad [\mathcal{L}] = M^4 \quad 7/19$$

The amazing success of the SM, so far (1974 → now)

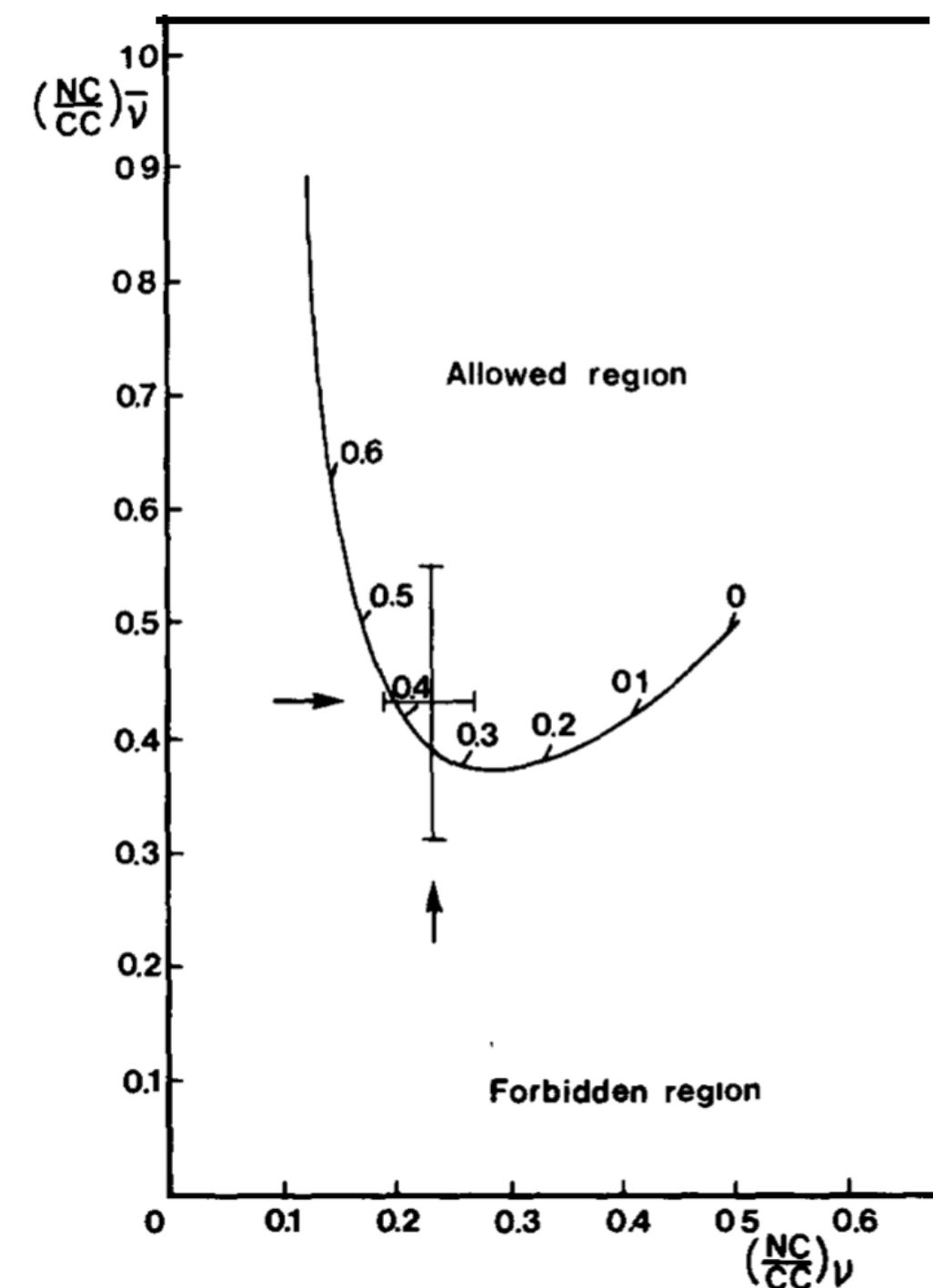
The discovery of neutral currents

1972 Weinberg $0.15 \leq \frac{d\sigma(\nu_\mu p \rightarrow \nu_\mu p)/dq^2}{d\sigma(\nu_\mu n \rightarrow \mu p)/dq^2} \leq 0.25$

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1974 CERN Gargamelle $\frac{NC}{CC}(\nu_\mu) = 0.22 \pm 0.04$



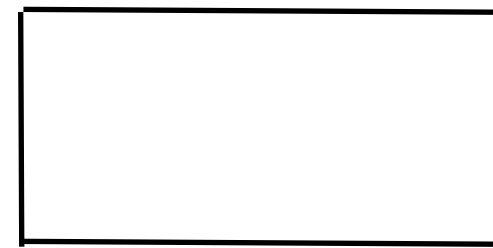
D. Haidt



Gargamelle emerita

Today exhibited on CERN ground

The progressive discovery of all the expected particles



$u(1968)$	$d(1968)$	$e(1897)$	$\nu_e(1956)$
	$s(1968)$	$\mu(1937)$	$\nu_\mu(1962)$

	$A_\mu(1905)$		
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In 1973, when all the ingredients of the SM were there, including CPV if 3 families,...

The progressive discovery of all the expected particles

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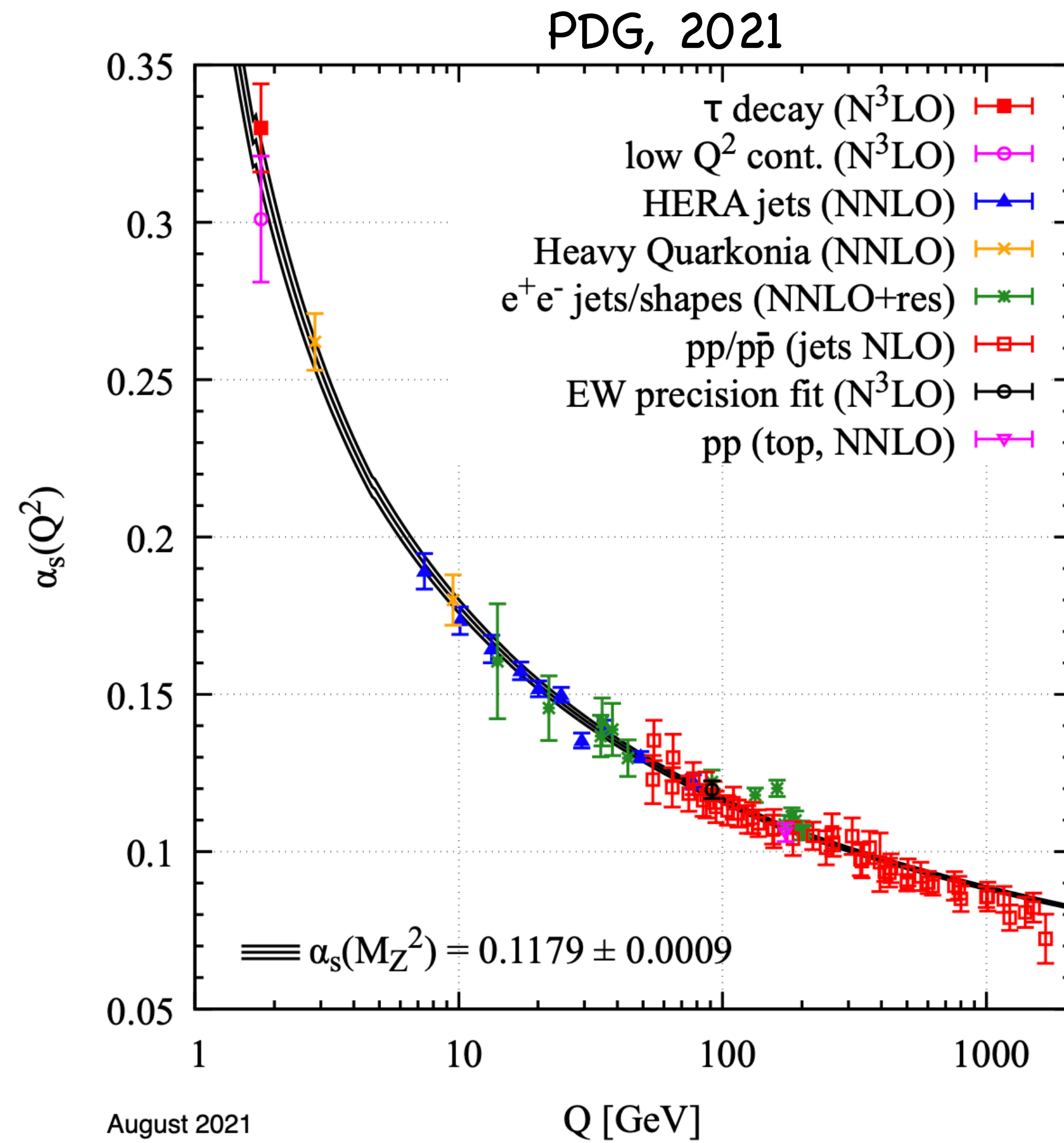
$h(2012)$

$u(1968)$	$d(1968)$	$e(1897)$	$\nu_e(1956)$
$c(1974)$	$s(1968)$	$\mu(1937)$	$\nu_\mu(1962)$
$t(1994)^*$	$b(1977)$	$\tau(1975)$	$\nu_\tau(2000)^*$

$G_\mu(1979)^*$	$A_\mu(1905)$	$W_\mu(1984)$	$Z_\mu(1984)$
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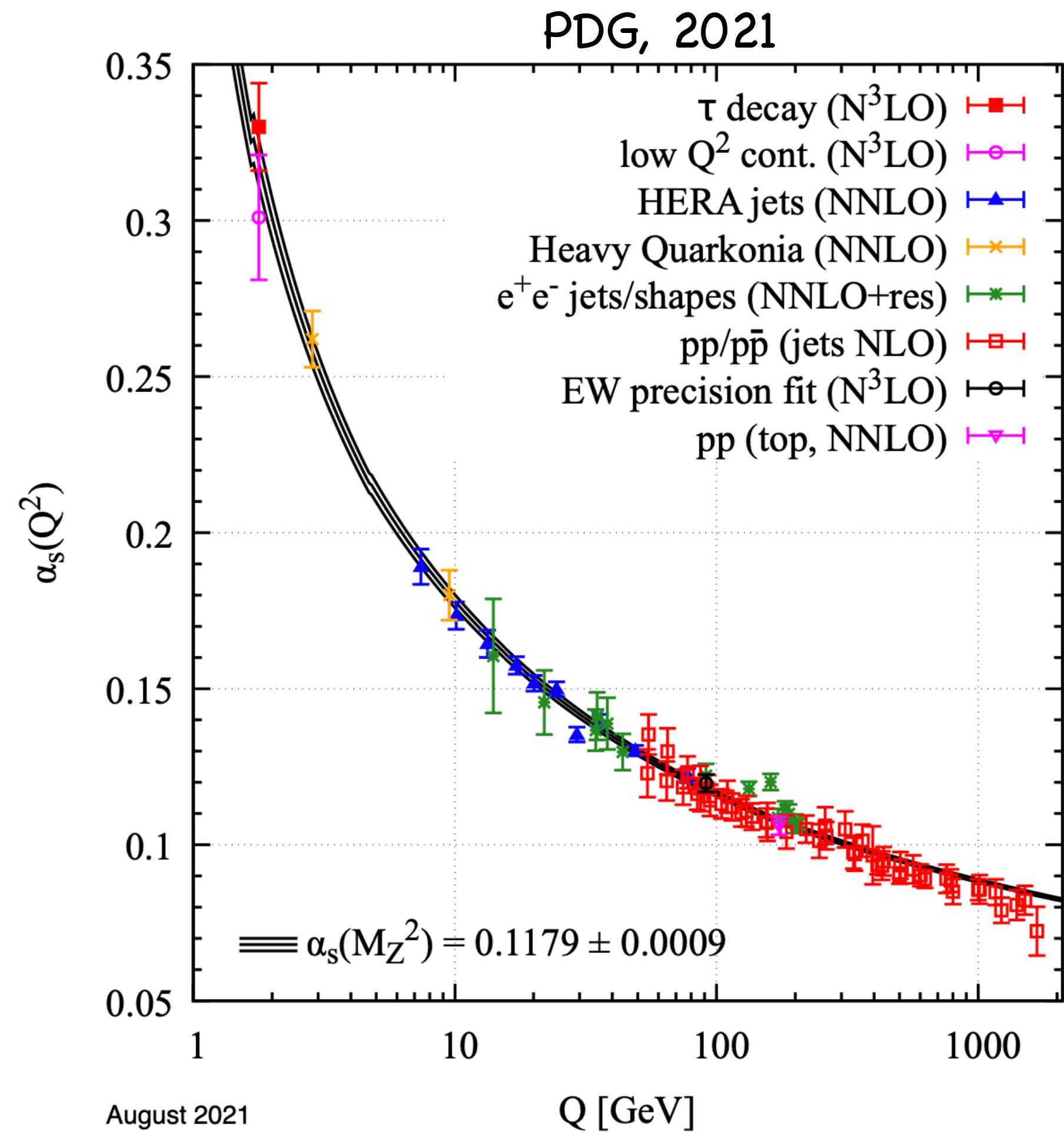
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Precision in QCD

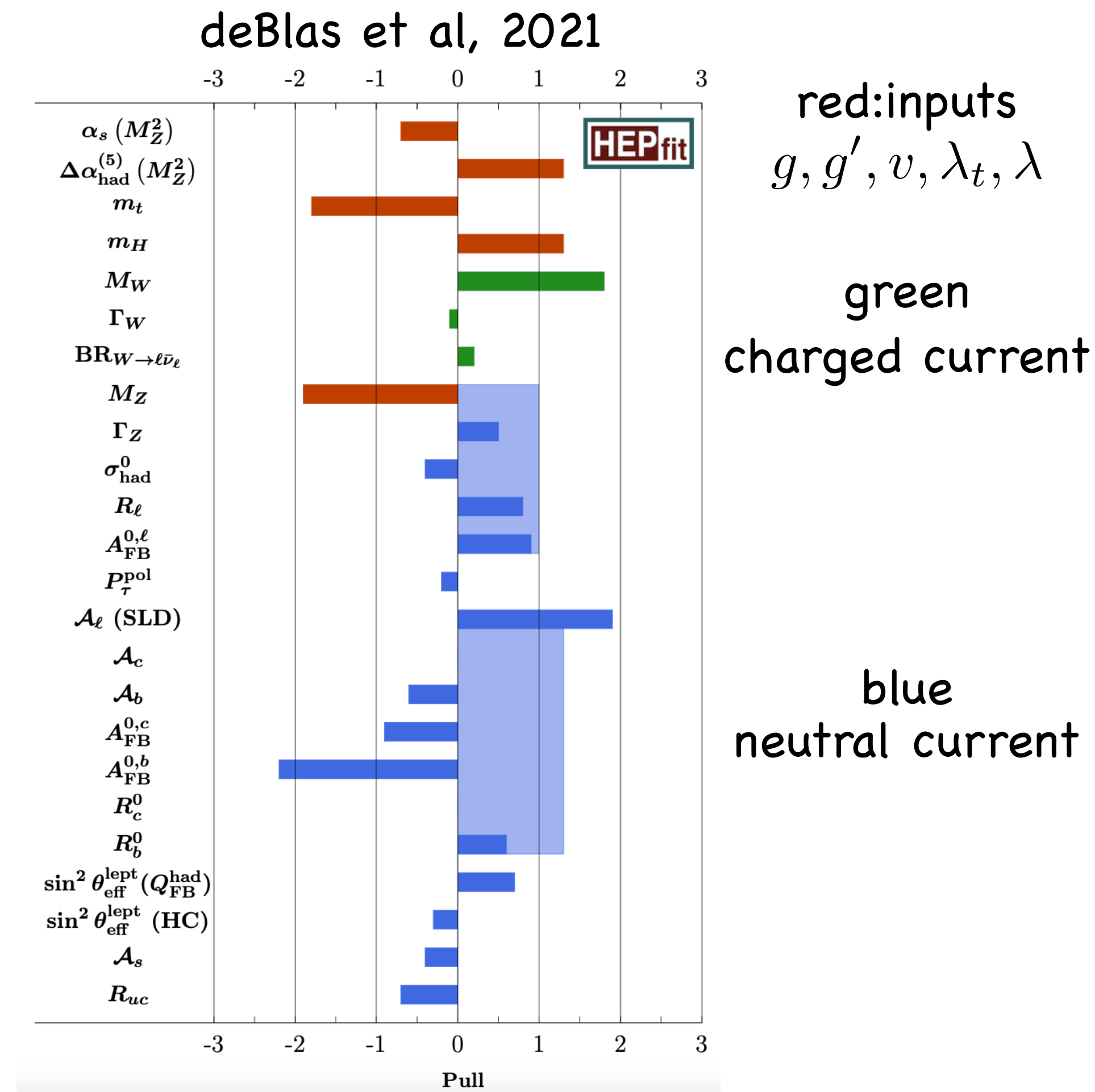


1977 Sterman, Weinberg

Precision in QCD and in the electroweak $SU(2) \times U(1)$

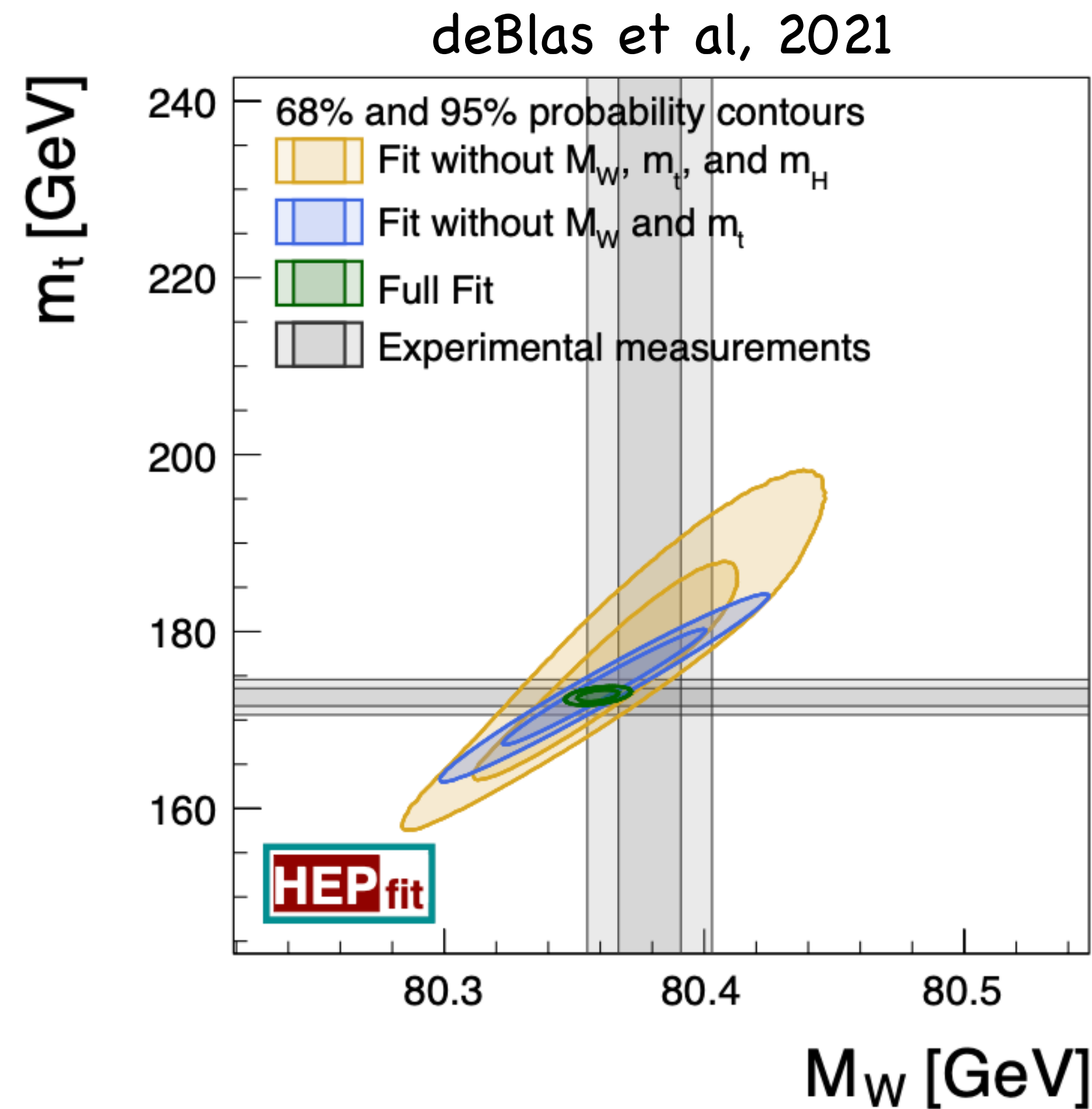


1977 Sterman, Weinberg



Each predicted observable removed from the fit

More on the precision of the “electroweak fit”

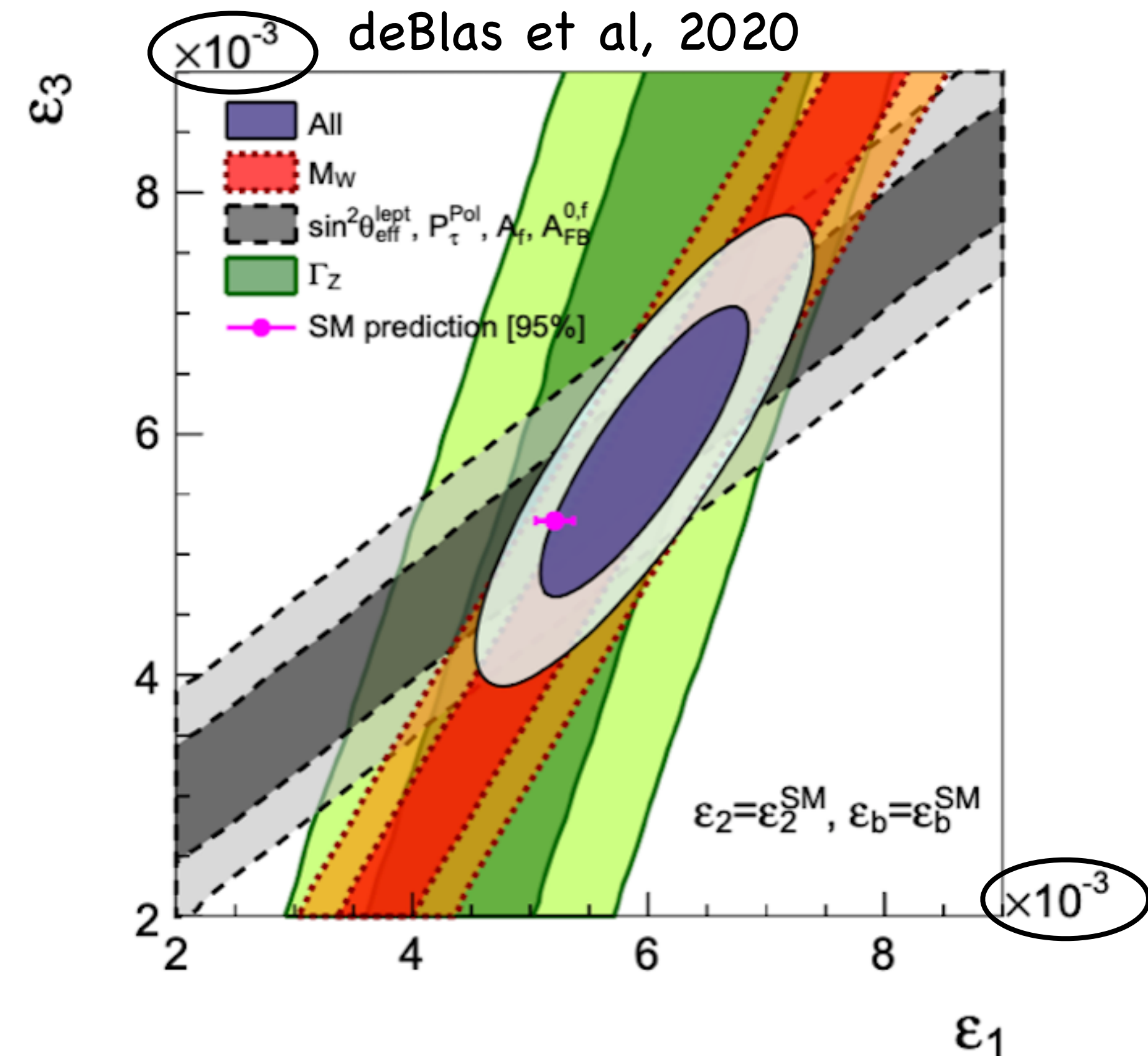
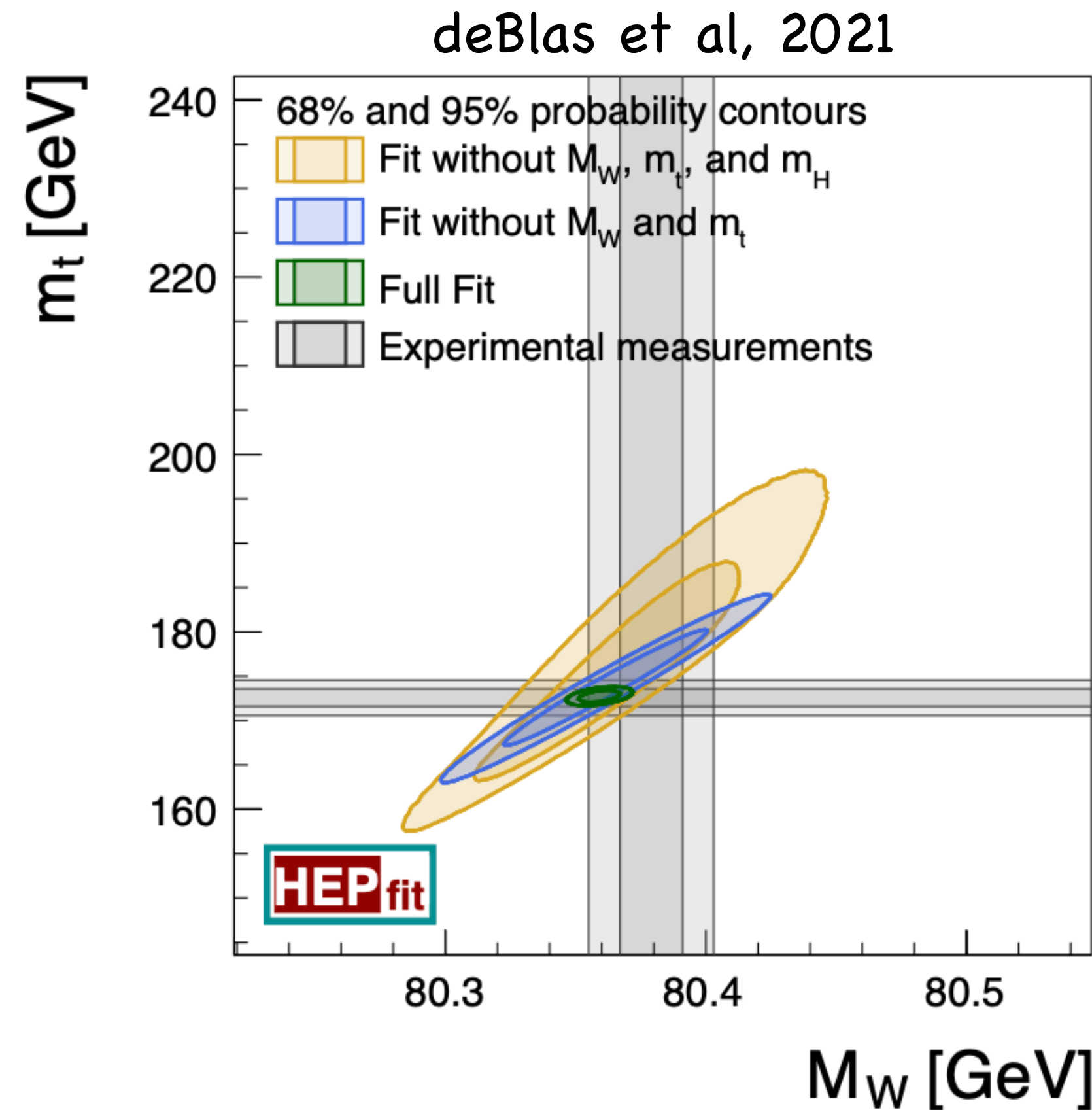


m_t and M_W NOT calculable in the SM

However...

1994	$m_t = 177 \pm 13^{+18}_{-19} \text{ GeV}$
2012	$m_H = 97^{+23}_{-17} \text{ GeV}$

More on the precision of the “electroweak fit”



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2012	$m_H = 97_{-17}^{+23} \text{ GeV}$

Two effective parameters from
all Z-pole obs.s and the W-mass

The “gauge sector” of the SM established!

The “ultimate” precision

$$a_e^{exp} = 1\,159\,652\,180.73(0.28) \times 10^{-12}$$

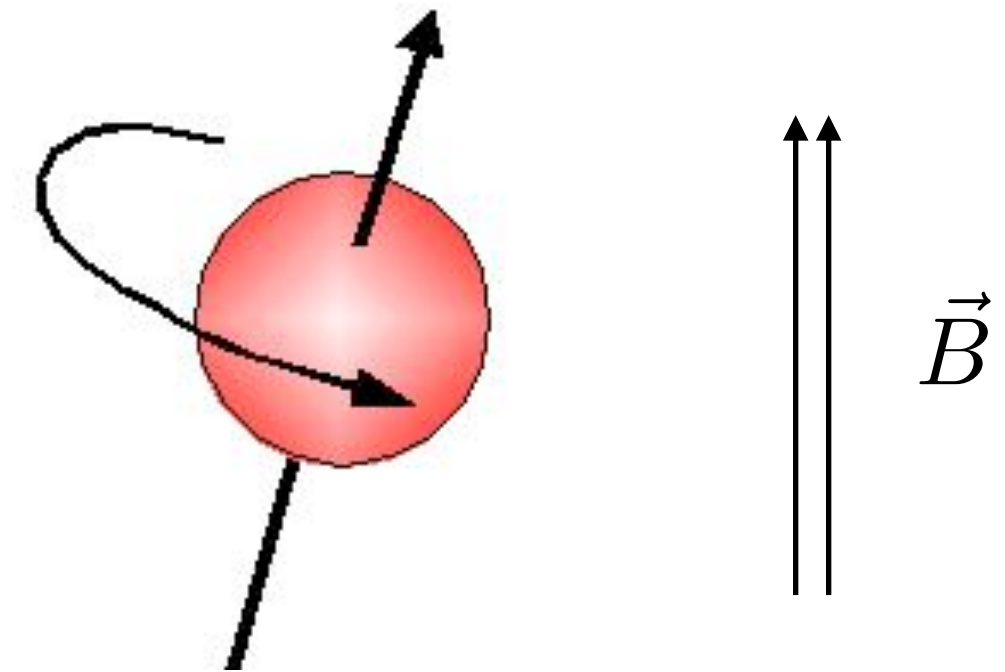
$$a_e^{th} = 1\,159\,652\,181.78(6)(4)(3)(77) \times 10^{-12}$$

0.7 ppb

$$a_\mu^{exp} = 1\,165\,920\,6.1(4.1) \times 10^{-10}$$

$$a_\mu^{th} = 1\,165\,918\,1.0(4.0)(1.8) \times 10^{-10}$$

0.4 ppm



1972 Jackiw, Weinberg

$$\Delta a_\mu(Weak) = \frac{G_F m_\mu^2}{\pi^2 \sqrt{2}} \left[\frac{5}{12} + \frac{4}{3} \left(\sin^2 \theta_W - \frac{1}{4} \right)^2 \right]$$

The synthetic nature of Particle Physics, once again

$$\begin{aligned}\mathcal{L}_{\sim SM} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\psi} \not{D}\psi & (\sim 1974-2000) \\ & + |D_\mu h|^2 - V(h) & (\sim 1990-2012\text{-now}) \\ & + \psi_i \lambda_{ij} \psi_j h + h.c. & (\sim 2000\text{- now})\end{aligned}$$

In () the approximate dates of the experimental confirmation of the various lines (at different levels)

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In () the approximate dates of the experimental tests
of the various lines (at different levels)

Weinberg: The world of physical phenomena reduced to a finite
set of fundamental equations/principles

The future of the Standard Model (now →)



A difference in the two sectors of the SM?

$$\mathcal{L}_{SM} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\Psi}\not{D}_{\mu}\Psi + |D_{\mu}\phi|^2$$

The “gauge sector”

$$+M^2|\phi|^2 - \lambda|\phi|^4 - \Lambda + \lambda_{ij}\phi\bar{\Psi}_i\Psi_j$$

The “Higgs sector”

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The “Higgs sector”

the hierarchy
problem

the CC problem

the flavour
problem

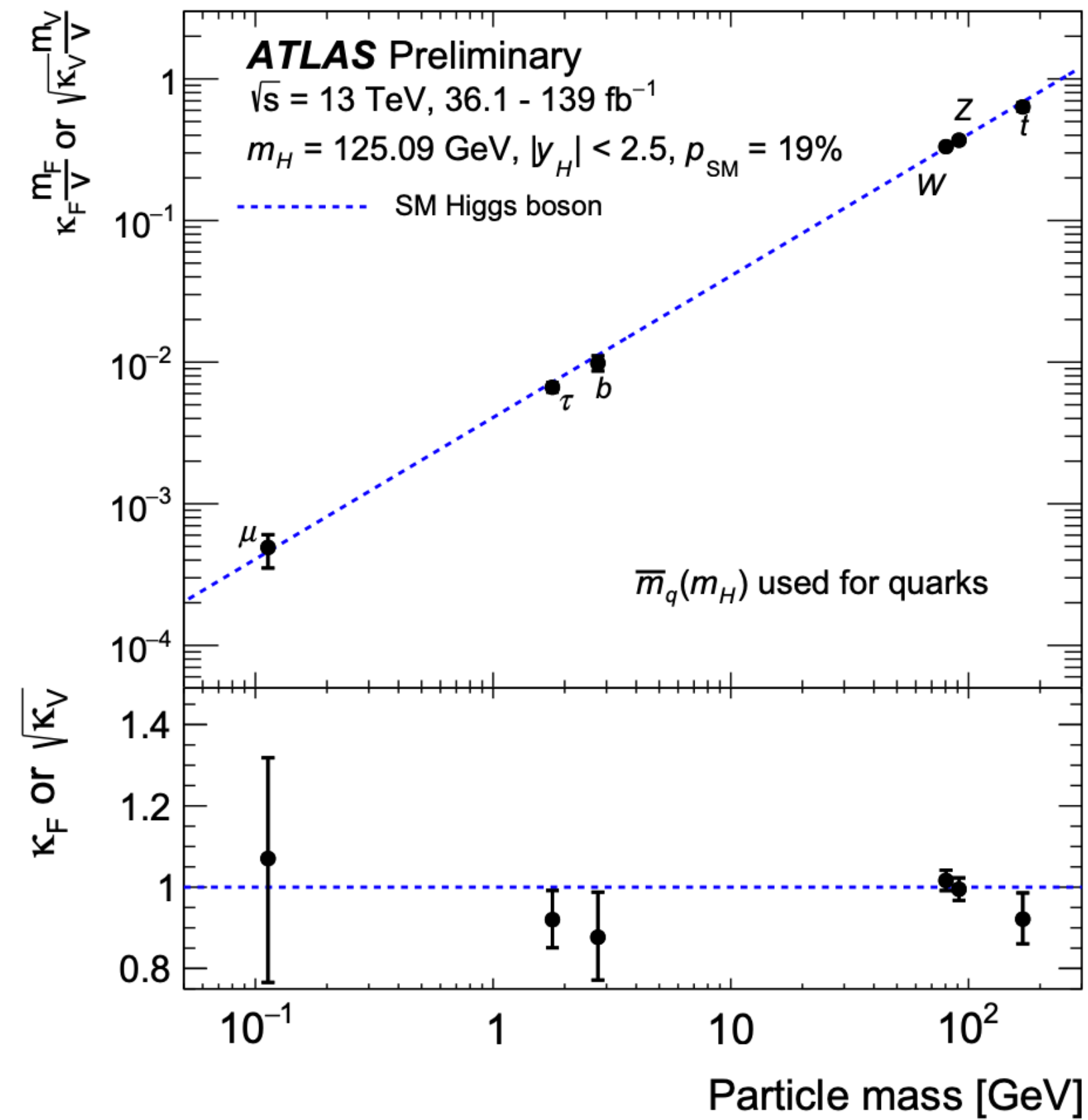
In EFT they look
much the same

No particle mass
calculable

Can we tell for sure that we know the true nature of EWSB?

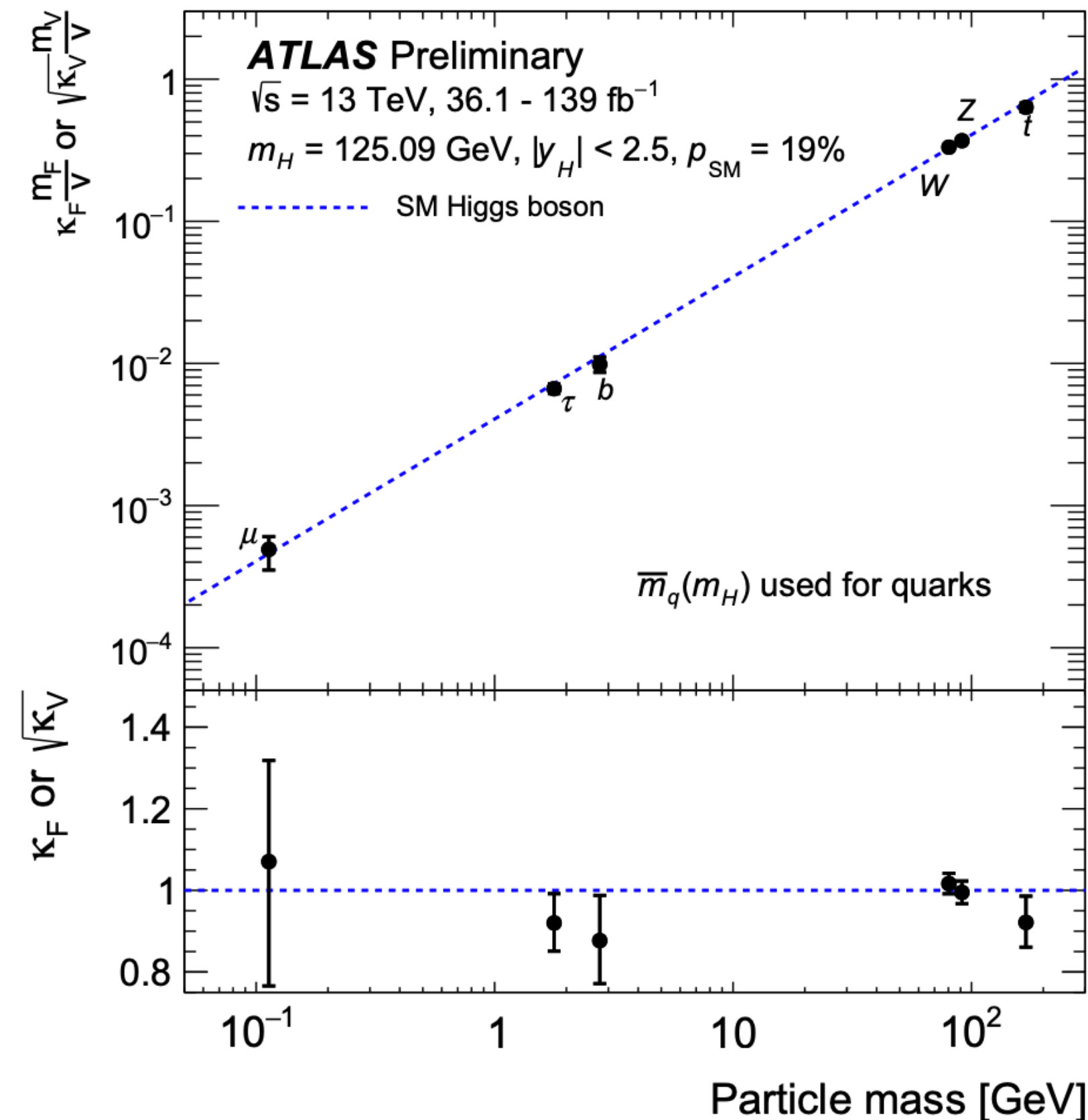
(a pretty conservative question, yes)

Comparing precision



H-couplings $(5 \div 20)\%$
 against EWPT $(0.1 \div 0.3)\%$

Comparing precision



This and the previous page make it
 that a “Higgs factory” is due...
 (Best if on the way to a higher energy collider)

What for?

- precision in H-couplings below 1%
- test H-couplings to the 2nd generation
- test H-self coupling
- test rare ($h \rightarrow Z\gamma, \tau\mu, \tau e, \mu e, CPV$)
and invisible H-decays
- ...

H-couplings $(5 \div 20)\%$
 against EWPT $(0.1 \div 0.3)\%$

Problems of (questions for) the SM

0. Which rationale for matter quantum numbers?

$$|Q_p + Q_e| < 10^{-21} e$$

GG, GQW 1974

1. Phenomena unaccounted for

neutrino masses
Dark matter

matter-antimatter asymmetry
inflation?

2. Why $\theta \lesssim 10^{-10}$?

$$\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Axions?

PQ 1977

W 1977

3. $\mathcal{O}_i : d(\mathcal{O}_i) \leq 4$ only?

neutrino masses
Are the protons forever?

W 1979

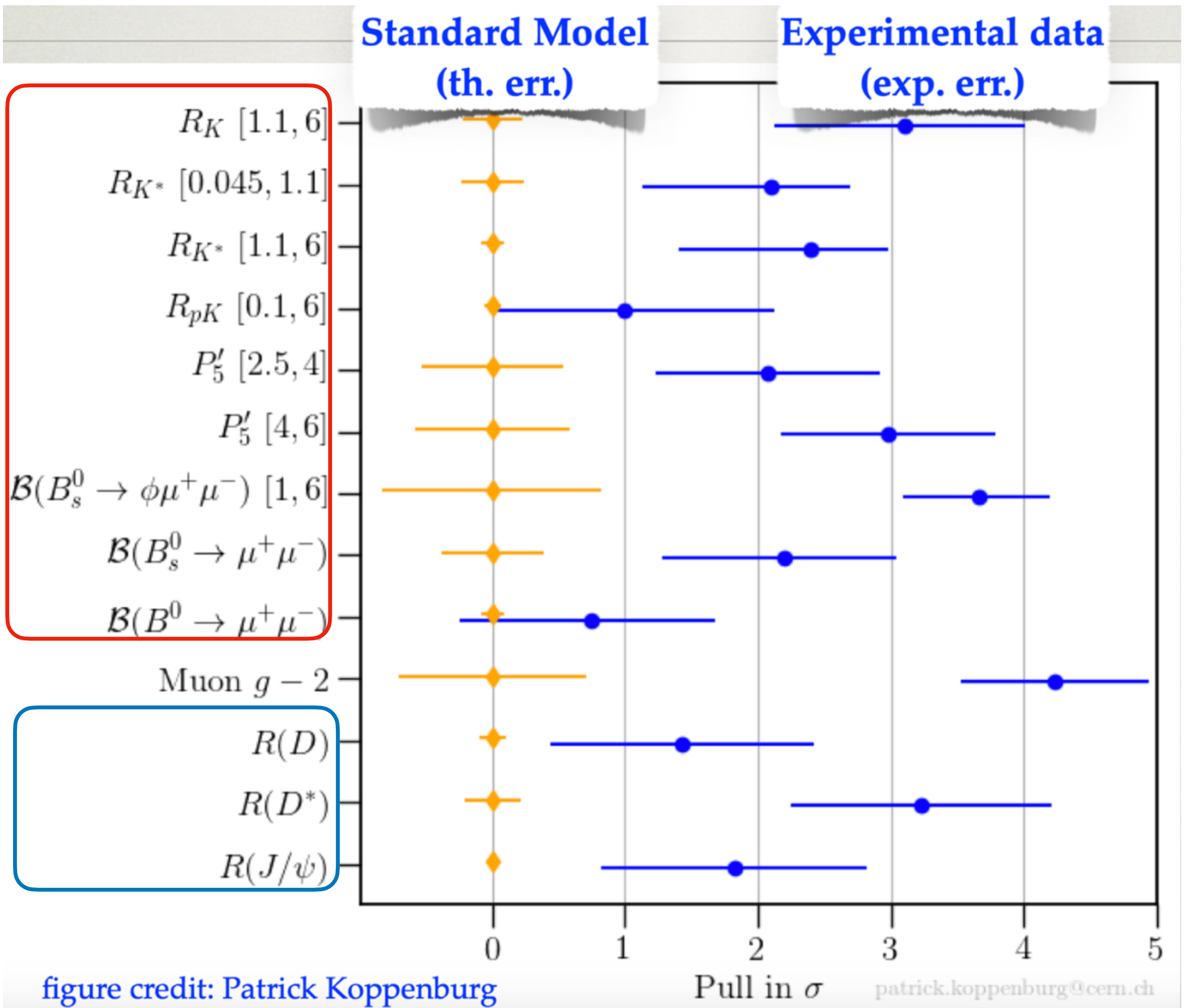
4. Lack of calculability

the hierarchy problem
the flavour problem

Current anomalies (a partial list)

Semi-leptonic B-decays

Neutral
Charged

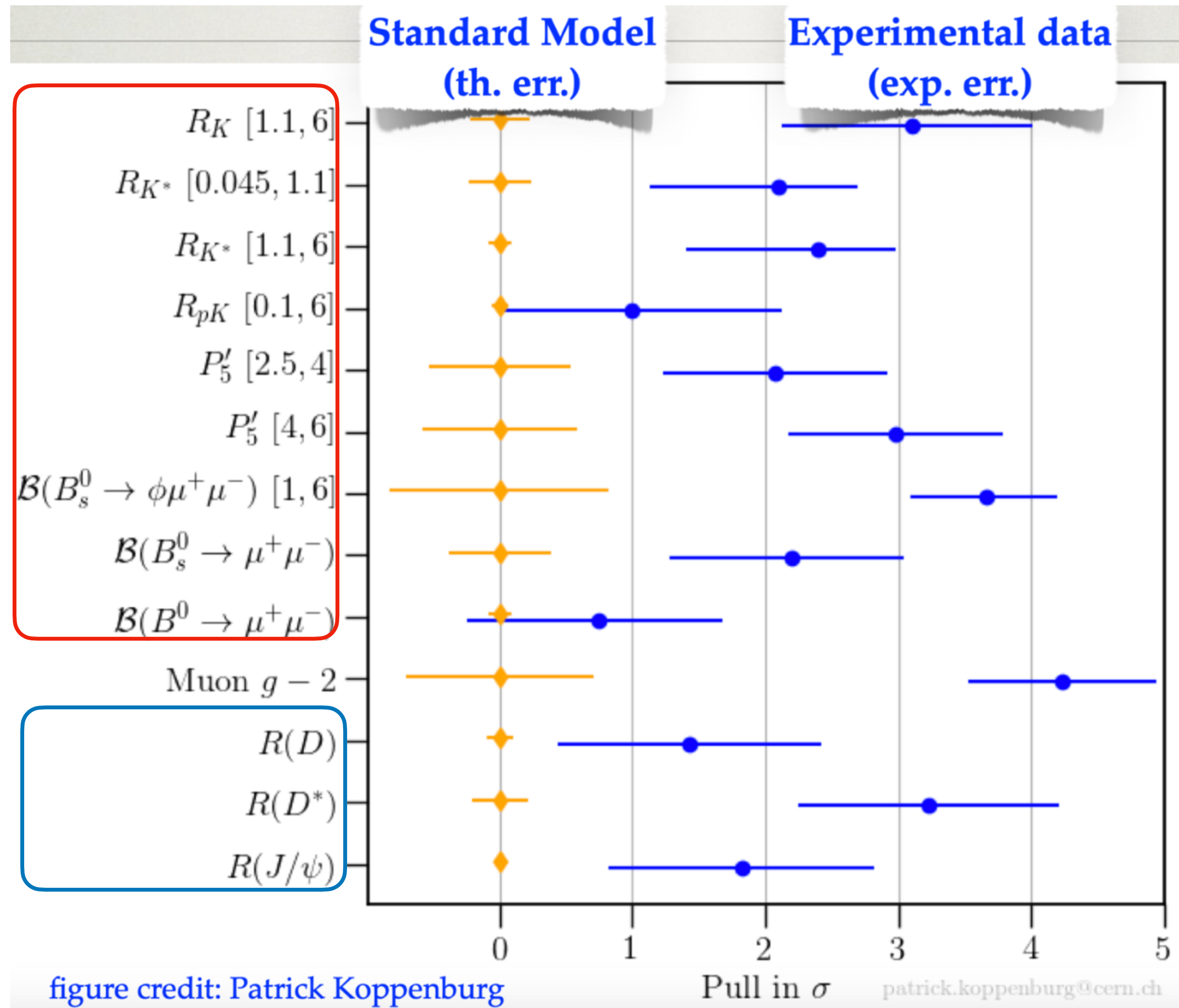


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Neutral

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General lessons:

- in many cases $\sigma_{th} \ll \sigma_{exp}$
- a precision programme to be followed in the coming years in flavour as in flavourless physics

If confirmed, in my view:

(not the subject of this talk)

- the coherence of B-decay anomalies suggestive of an approximate $U(2)^n$ flavour symmetry
- Δa_μ "easily" interpreted as a sign of supersymmetry

If they are roses... they will flourish

Where is the scale of flavour?

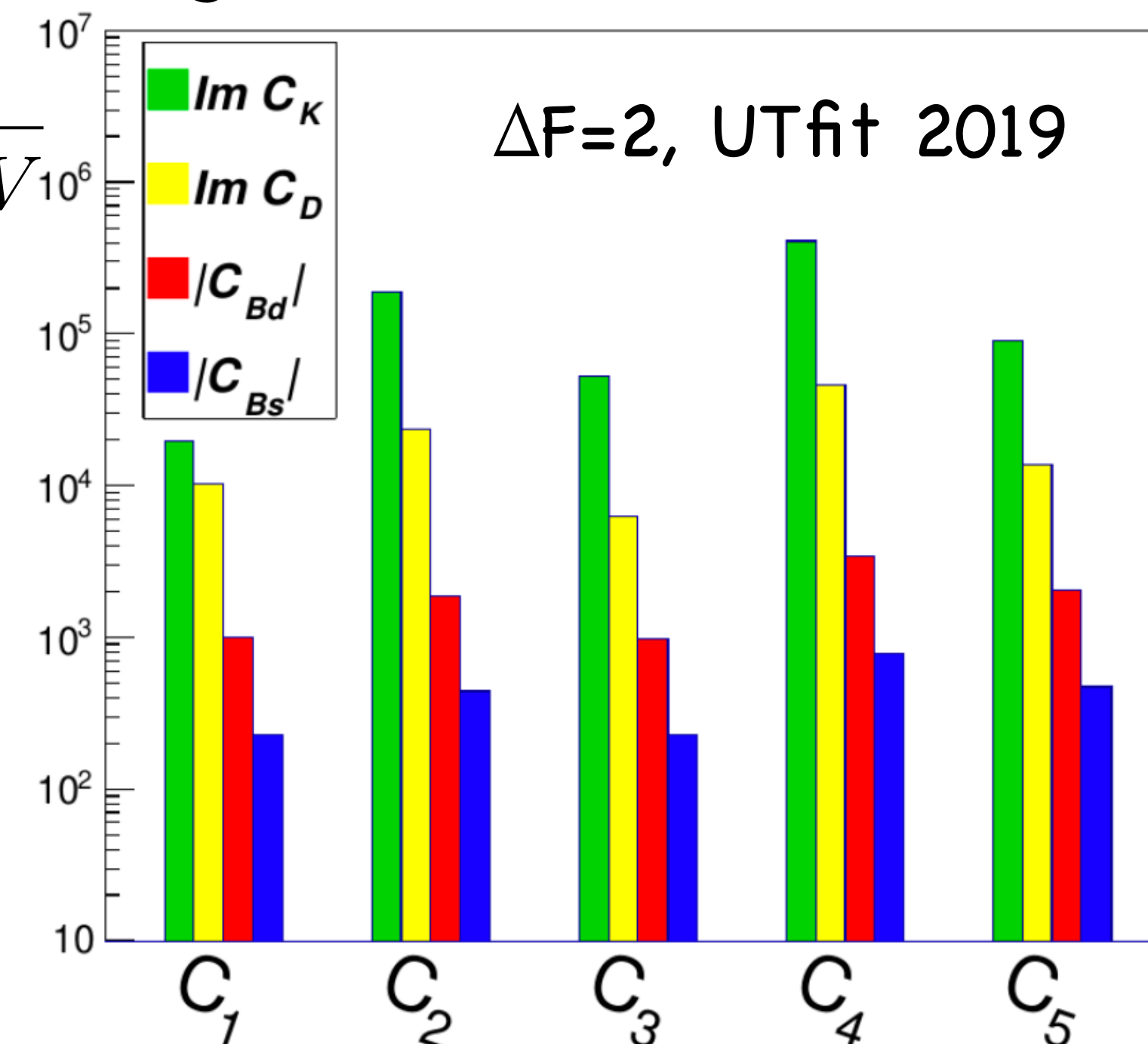
1. Flavour physics confined to high energy

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i^\alpha \frac{C_i^\alpha}{(\Lambda_i^\alpha)^2} (\bar{f} f \bar{f} f)_i^\alpha \quad \frac{\Lambda^\alpha}{\sqrt{C^\alpha} \text{TeV}}$$

$i = 1, \dots, 5$ = different Lorentz structures

$\alpha = K, D, B_d, B_s$

or



2. New physics in the multiTeV hidden by a suitable (approximate) flavour symmetry

Let us make Weinberg speak

From NYRB, November 7, 2012 “Physics: What We Do and Don’t Know”

“Even so, the Standard Model is clearly not the final theory. Its equations involve a score of numbers, like the masses of quarks, that have to be taken from experiment without our understanding why they are what they are. [...] Further, the SM does not include the longest-known and most familiar force, the force of gravitation”

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A few paragraphs later

"Inflation is naturally chaotic. Bubbles form in the expanding universe [...] perhaps each with different values for what we usually call the constants of nature.

If this is true, then the hope of finding a rational explanation for the precise values of quark masses and other constants of the SM that we observe in our big bang is doomed. [...] We would have to content ourselves with a crude anthropic explanation of some aspects of the universe we see.

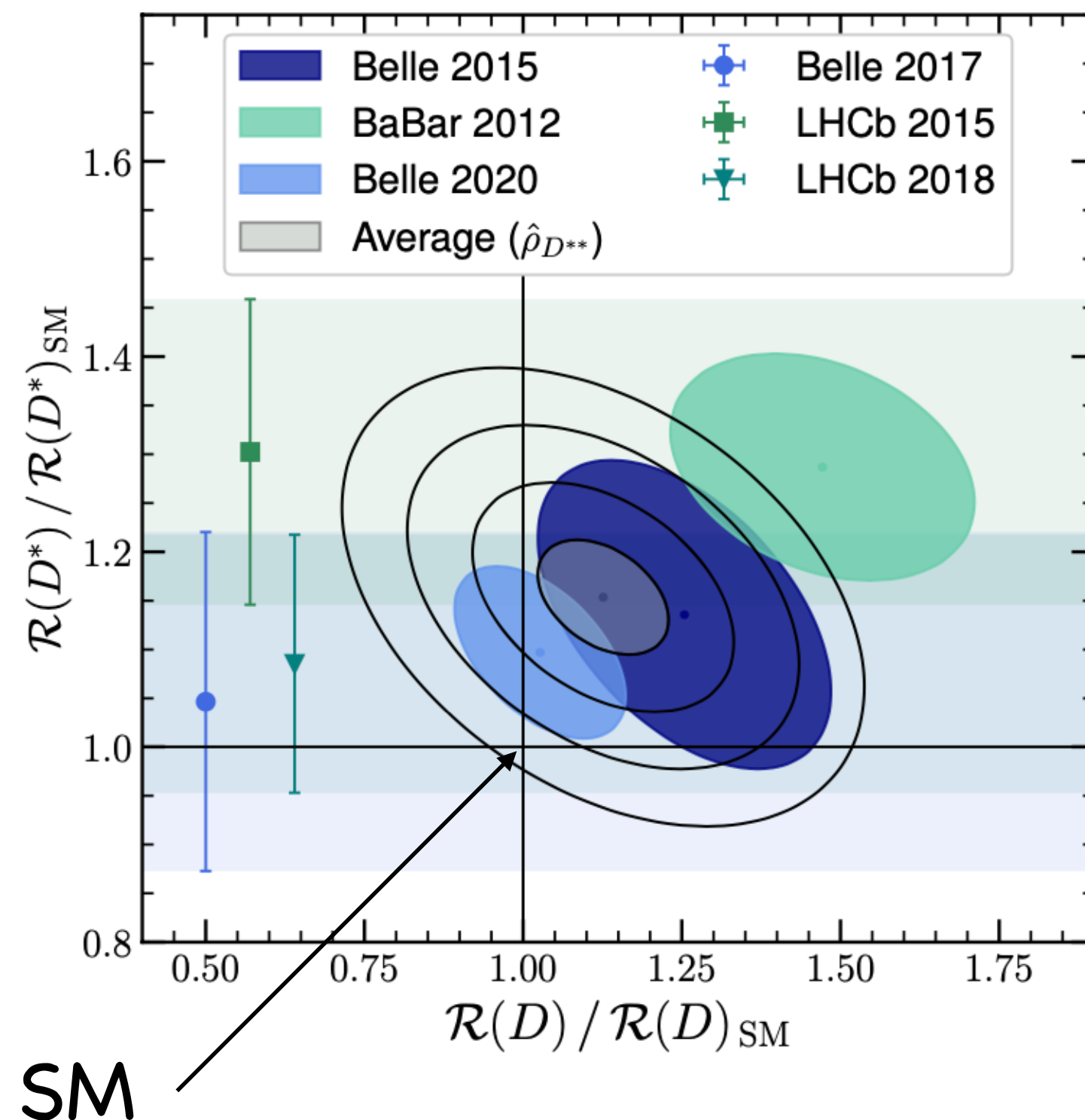
So far, this anthropic speculation seems to provide the only explanation of the observed value of the dark energy."

Backup

A violation of Lepton Flavour Universality?

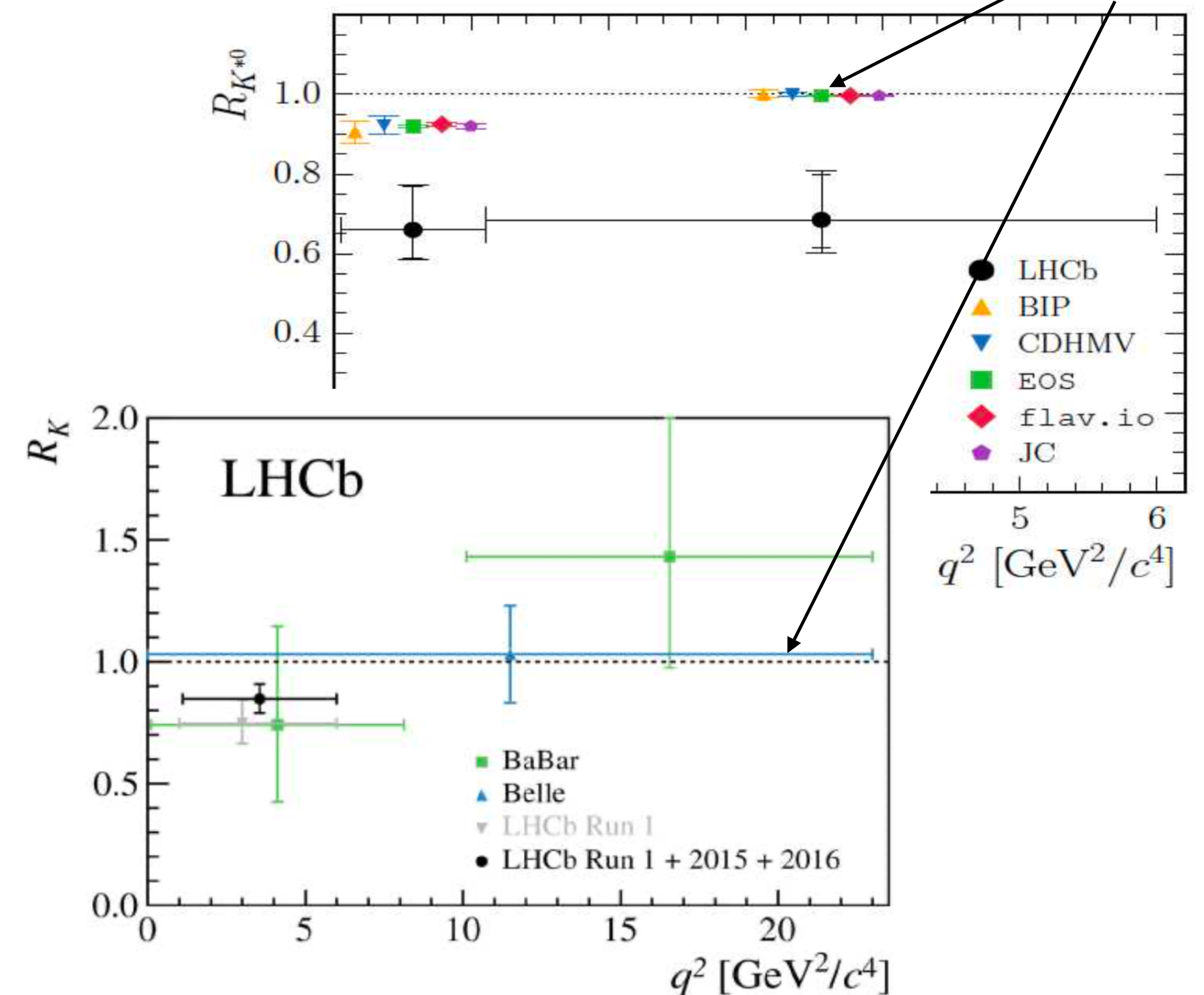
$$R_{D^{(*)}} = \frac{BR(B \rightarrow D^{(*)}\tau\nu)}{BR(B \rightarrow D^{(*)}l\nu, l = \mu, e)}$$

Bernlochner et al 2021



Yet too early to say

$$R_{K^{(*)}} = \frac{BR(B \rightarrow K^{(*)}\mu\mu)}{BR(B \rightarrow K^{(*)}ee)}$$



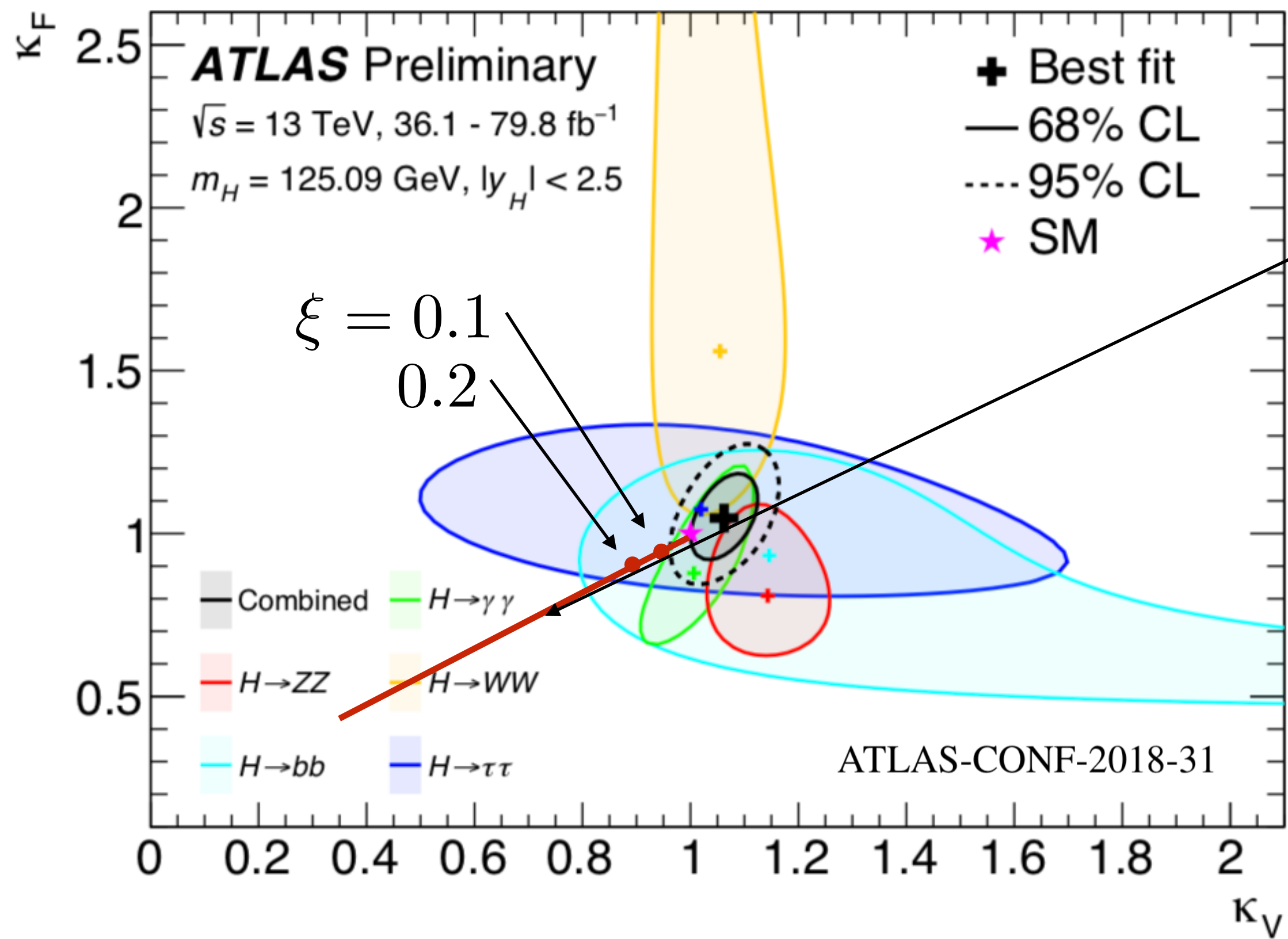
Still in the limbo, but

Observable	Current LHCb	LHCb 2025	Upgrade II
EW Penguins			
$R_K (1 < q^2 < 6 \text{ GeV}^2 c^4)$	0.1 [4]	0.025	0.007
$R_{K^*} (1 < q^2 < 6 \text{ GeV}^2 c^4)$	0.1 [5]	0.031	0.008
$b \rightarrow c \ell^- \bar{\nu}_\ell$ LUV studies			
$R(D^*)$	0.026 [15, 16]	0.0072	0.002
$R(J/\psi)$	0.24 [17]	0.071	0.02

the expected future precision will settle the issue

Higgs couplings

$$\mathcal{L} = g_f k_F H \bar{f} f + g_V k_V V_\mu H^+ \partial_\mu H$$



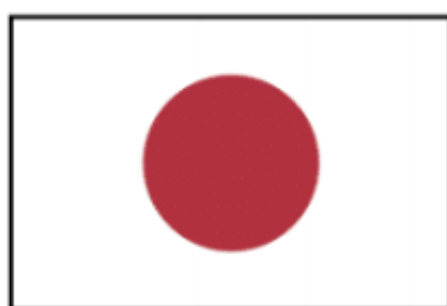
$$\xi = \frac{v^2}{f^2} \qquad k_F = k_V = \sqrt{1 - \xi}$$

Now $f \gtrsim 600 \div 800 \text{ GeV}$

$$\xi \approx \frac{g_*^2 v^2}{m_*^2}$$

Collider	Energy	Luminosity	ξ [1σ]
LHC	14 TeV	300 fb^{-1}	$6.6 - 11.4 \times 10^{-2}$
LHC	14 TeV	3 ab^{-1}	$4 - 10 \times 10^{-2}$

$$f \rightarrow 1 \div 1.5 \text{ TeV}$$



How can we reach these goals?

	T ₀				+5				+10					+15				+20			...	+26
ILC	0.5/ab 250 GeV					1.5/ab 250 GeV					1.0/ab 500 GeV				0.2/ab 2m _{top}	3/ab 500 GeV						
CEPC	5.6/ab 240 GeV						16/ab M _Z	2.6 /ab 2M _W													SppC =>	
CLIC	1.0/ab 380 GeV								2.5/ab 1.5 TeV							5.0/ab => until +28 3.0 TeV						
FCC	150/ab ee, M _Z		10/ab ee, 2M _W		5/ab ee, 240 GeV			1.7/ab ee, 2m _{top}										hh,eh =>				
LHeC	0.06/ab					0.2/ab					0.72/ab											
HE-LHC	10/ab per experiment in 20y																					
FCC eh/hh	20/ab per experiment in 25y																					



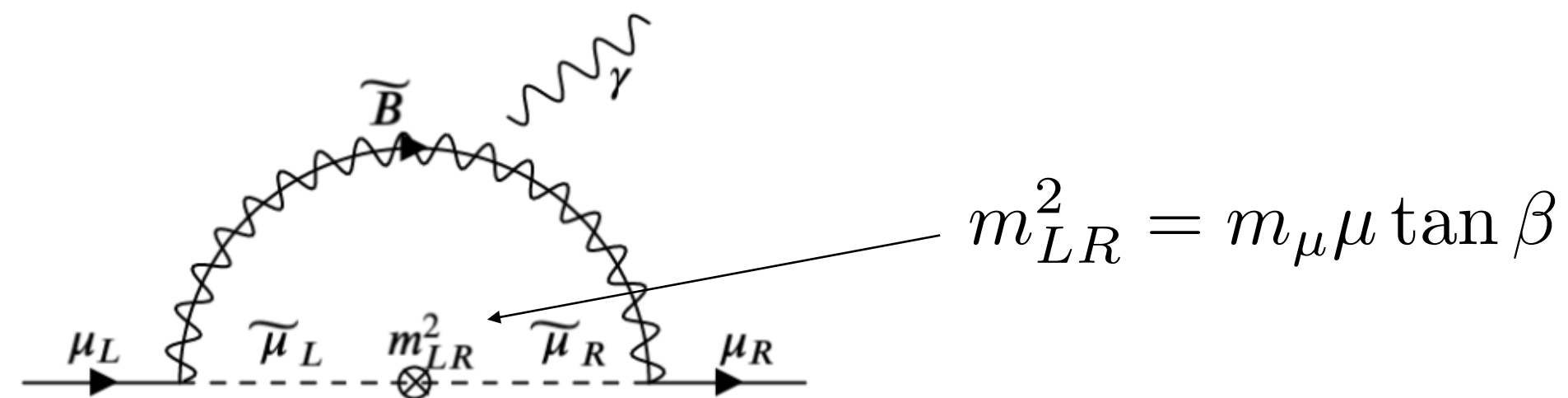
IF Δa_μ confirmed

SUSY as an "EASY" and "MOTIVATED"

1. Relevant (low energy)

$$M_1 \tilde{b} \tilde{b}, \quad M_2 \tilde{w} \tilde{w}, \quad m_{\tilde{\mu}_L} \tilde{\mu}_L^+ \tilde{\mu}_L, \quad m_{\tilde{\mu}_R} \tilde{\mu}_R^+ \tilde{\mu}_R, \quad \mu H_1 H_2, \quad \tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$$

2. Size of the effect:



$$\Delta a_\mu|_{SUSY} \approx \frac{g_1^2}{16\pi^2} m_\mu^2 \frac{M_1 \mu \tan \beta}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} \approx 2.5 \cdot 10^{-9} \left(\frac{\tan \beta}{10} \right) \left(\frac{\mu}{1 \text{ TeV}} \right) \left(\frac{M_1}{100 \text{ GeV}} \right) \left(\frac{200 \text{ GeV}}{m_{\tilde{\mu}_L}} \right)^2 \left(\frac{200 \text{ GeV}}{m_{\tilde{\mu}_R}} \right)^2$$

3. Main constraints on parameter space (high energy)

- No coloured partners below $1 \div 2 \text{ TeV}$

$$M_3 \tilde{g} \tilde{g}, \quad M_3 \gtrsim a \text{ few TeV}$$

- No flavour violations $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \text{etc}$

$$m_{\tilde{e}_L} = m_{\tilde{\mu}_L} = m_{\tilde{\tau}_L} \quad (\text{and similarly for } m_{\tilde{l}_R})$$

Due to $m_{LR}^2(\tau) = m_\tau \mu \tan \beta \approx (150 \text{ GeV})^2 \frac{\tan \beta}{10} \frac{\mu}{\text{TeV}} \Rightarrow \tilde{\tau} = \text{lightest s-lepton}$

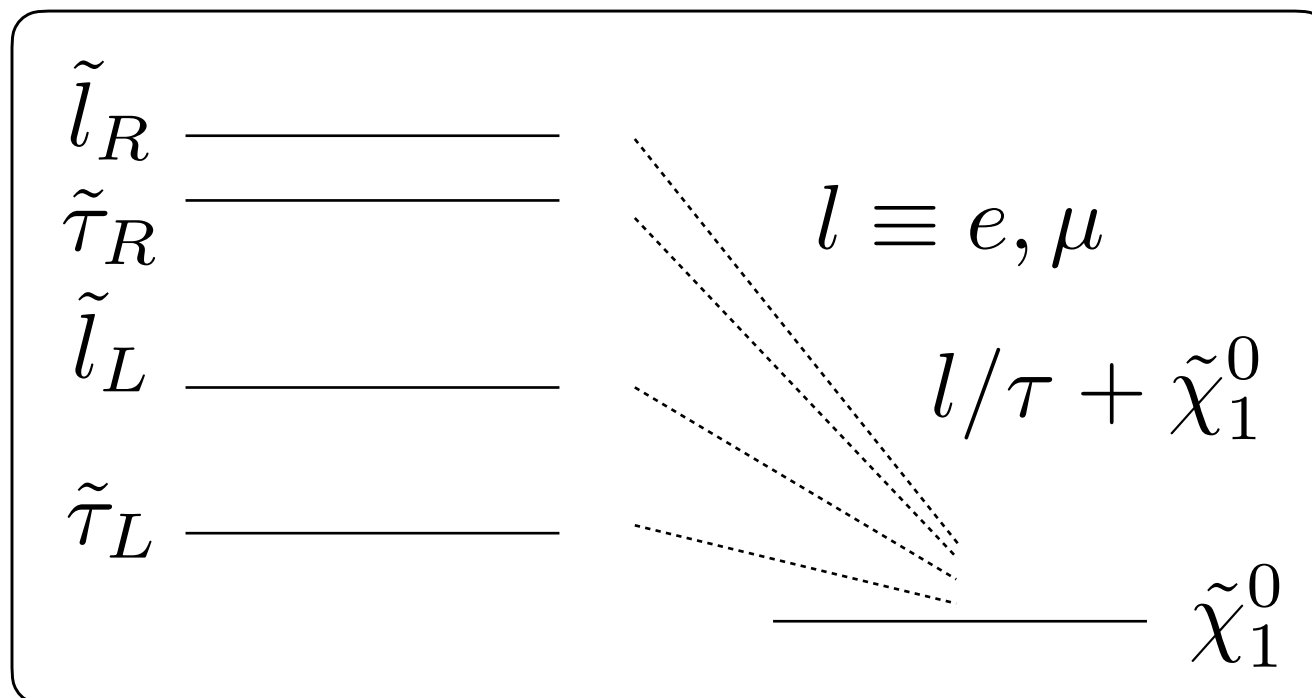
- Cancellations needed

$$m_Z^2 = -2(m_{H_2}^2 + |\mu|^2)$$

- If non-zero phases, from $d_e < 1.2 \cdot 10^{-29} e \cdot \text{cm}$

$$m_{\tilde{\nu}_{eL}} \gtrsim 40 \text{ TeV} (\sin \phi_\mu \tan \beta)^{1/2}$$

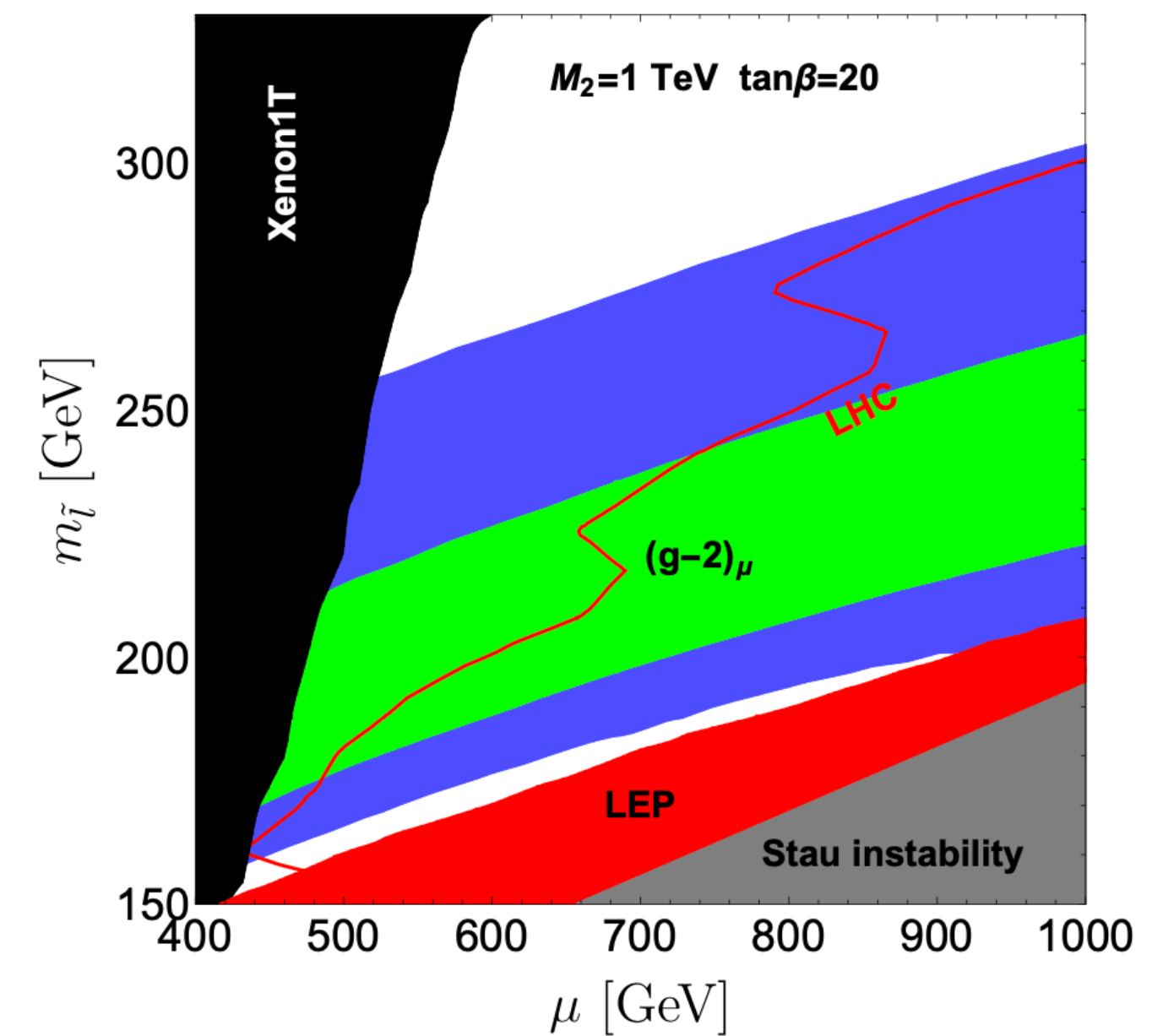
4. Direct signals



$$M_1 < m_{\tilde{l}_L}, m_{\tilde{l}_R} < M_2 < \mu < M_3$$

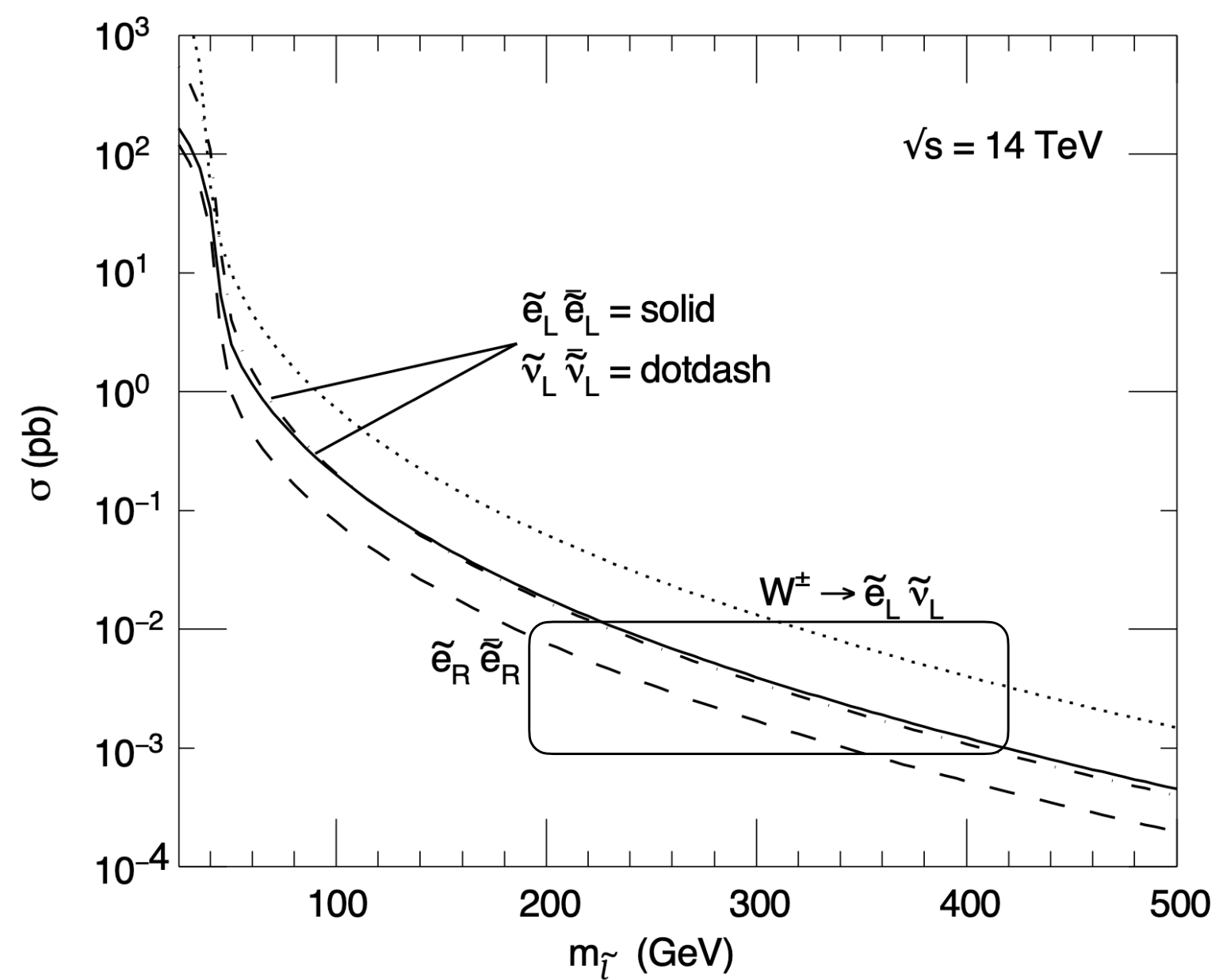
- **LHC** $\sigma(pp \rightarrow \tilde{l}\tilde{l}) \approx 1 \div 10 \text{ fb}$
backgrounds: $VV, V + jets, V^* \rightarrow l\bar{l}, t\bar{t}, t + V$
- **DM** $\tilde{\chi}_1^0 \tilde{\tau}$ co-annihilation $\Delta m \equiv m_{\tilde{\tau}} - m_{\tilde{\chi}_1^0} \lesssim 15 \text{ GeV}$

Cox et al, 2021



green: a_μ at 1σ

blue: a_μ at 2σ



Baer et al, 1993

