

p_T fluctuations and multiparticle clusters in heavy-ion collisions

(“CLUSTER SCALING”)

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Based very closely on WB, Hiller, Florkowski, Bożek, [nucl-th/0510033](#),
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A simple way of doing the statistics

- - variable dividing into classes (multiplicity n), dynamical variable $p_i = |\vec{p}_{T,i}|$
- - divide the events into classes of **the same** n , $P(n) = N(n)/N_{\text{all}}$ is the probability of obtaining event of multiplicity n
- - n and $p_1, p_2, \dots, p_n$ vary from event to event. The probability of a given configuration is $P(n)\rho_n(p_1, \dots, p_n)$, where $\rho_n(p_1, \dots, p_n)$ is the conditional probability distribution of occurrence of $p_1, \dots, p_n$ provided we have multiplicity n . Note ρ_n depends functionally on n . The normalization is

$$\sum_n P(n) = 1, \quad \int dp_1 \dots dp_n \rho_n(p_1, \dots, p_n) = 1$$

The *marginal* probability densities are defined as

$$\rho_n^{(n-k)}(p_1, \dots, p_{n-k}) \equiv \int dp_{n-k+1} \dots dp_n \rho_n(p_1, \dots, p_n),$$

with $k = 1, \dots, n - 1$. These are also normalized to 1

- We introduce

$$\langle p \rangle_n \equiv \int dp \rho_n(p) p, \quad \text{var}_n(p) \equiv \int dp \rho_n(p) (p - \langle p \rangle_n)^2,$$

$$\text{cov}_n(p_1, p_2) \equiv \int dp_1 dp_2 (p_1 - \langle p \rangle_n) (p_2 - \langle p \rangle_n) \rho_n(p_1, p_2).$$

The subscript n indicates that **the averaging is taken within samples of a given multiplicity n**

- - Remark: note that I am not using the **inclusive** quantities, defined through the **inclusive** probability distributions related to the marginal probability distributions in the following way:

$$\rho_{\text{in}}(x) \equiv \sum_n P(n) \int dp_1 \dots dp_n \sum_{i=1}^n \delta(x - p_i) \rho_n(p_1, \dots, p_n) = \sum_n n P(n) \rho_n(x),$$

$$\begin{aligned} \rho_{\text{in}}(x, y) &\equiv \sum_n P(n) \int dp_1 \dots dp_n \sum_{i,j=1, j \neq i}^n \delta(x - p_i) \delta(y - p_j) \rho_n(p_1, \dots, p_n) \\ &= \sum_n n(n-1) P(n) \rho_n(x, y) \end{aligned}$$

which are normalized to $\langle n \rangle$ and $\langle n(n-1) \rangle$, respectively

- - Define wider classes, $1 \leq n_1 \leq n \leq n_2$, $\sum_n = \sum_{n=n_1}^{n_2}$
- - For the variable $M = \sum_{i=1}^n p_i/n$ we find immediately an *exact* result

$$\langle M \rangle = \sum_n P(n) \int dp_1 \dots dp_n M \rho_n(p_1, \dots, p_n) = \sum_n P(n) \langle p \rangle_n,$$

$$\langle M^2 \rangle = \sum_n P(n) \int dp_1 \dots dp_n M^2 \rho_n(p_1, \dots, p_n)$$

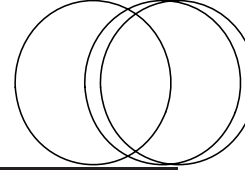
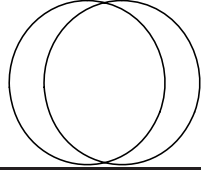
$$= \sum_n \frac{P(n)}{n} \langle p^2 \rangle_n + \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1, j \neq i}^n \text{cov}_n(p_i, p_j) + n(n-1) \langle p \rangle_n^2 \right]$$

$$\sigma_M^2 = \sum_n \frac{P(n)}{n} \sigma_{p,n}^2 + \sum_n P(n) (\langle p \rangle_n)^2 - \left(\sum_n P(n) \langle p \rangle_n \right)^2 +$$

$$+ \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1, j \neq i}^n \text{cov}_n(p_i, p_j) \right]$$

A look at some data: PHENIX @ 130 GeV

$$|\eta| < 0.35, 0.2 < p_T < 1.5 \text{ GeV}, \Delta\phi = 45^\circ$$



centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
$\langle M \rangle$	523	523	523	520
σ_p	290	290	290	289
σ_M	38.6	41.1	49.8	61.1
$\langle M \rangle^{\text{mix}}$	523	523	523	520
σ_M^{mix}	37.8	40.3	48.8	60.0

PHENIX, PRC66 (2002) 024901, nucl-ex/0203015

$\langle M \rangle$ and σ_p are practically **constant** in the “fiducial” centrality range $c = 0 - 30\%$ **(1)**

(1) allows us to replace $\langle p \rangle_n$ with $\langle M \rangle$ and $\sigma_{p,n}^2 = \langle p^2 \rangle_n - \langle p \rangle_n^2$ with σ_p^2 :

$$\sigma_M^2 = \sigma_p^2 \sum_n \frac{P(n)}{n} + \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1, j \neq i}^n \text{cov}_n(p_i, p_j) \right]$$

[corrections can be worked out]

In mixed events, by construction, particles are not correlated, hence the covariance term vanishes and

$$\sigma_M^{2,\text{mix}} = \sigma_p^2 \sum_n \frac{P(n)}{n} \simeq \sigma_p^2 \left(\frac{1}{\langle n \rangle} + \frac{\sigma_n^2}{\langle n \rangle^3} + \dots \right) \quad (2)$$

where we have used the fact that $P(n)$ is narrow and expanded $1/n = 1/[\langle n \rangle + (n - \langle n \rangle)]$ to second order in $(n - \langle n \rangle)$

centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
σ_p	290	290	290	289
σ_M^{mix}	37.8	40.3	48.8	60.0
$\sigma_p \sqrt{\frac{1}{\langle n \rangle} + \frac{\sigma_n^2}{\langle n \rangle^3}}$	38.2	40.5	49.8	60.8

(2) works within 1% [which is a trivial statistical statement checking that our approximations work]

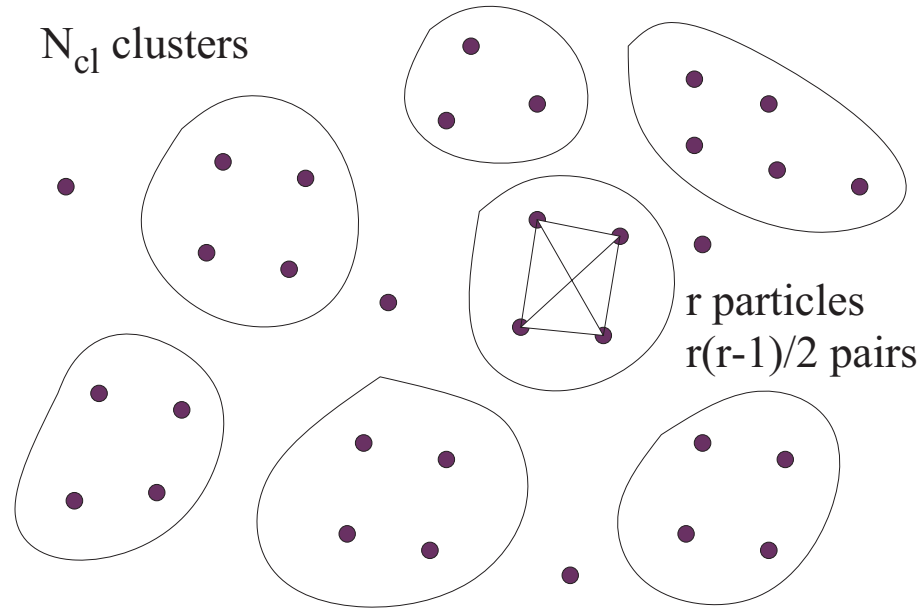
Subtracting the last two equations yields

$$\sigma_{\text{dyn}}^2 \equiv \sigma_M^2 - \sigma_M^{\text{mix},2} = \sum_n \frac{P(n)}{n^2} \sum_{i,j=1, j \neq i}^n \text{cov}_n(p_i, p_j) \simeq \frac{1}{\langle n \rangle^2} \sum_{i,j=1, j \neq i}^{\langle n \rangle} \text{cov}(p_i, p_j) \quad (3)$$

centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_M	38.6	41.1	49.8	61.1
σ_M^{mix}	37.8	40.3	48.8	60.0
$\sigma_{\text{dyn}} \sqrt{\langle n \rangle}$	60.3 ± 1.6	59.2 ± 1.5	59.8 ± 1.2	57.7 ± 1.1

$\sigma_{\text{dyn}}^2 \sim 1/\langle n \rangle$ (within 2%, round-off errors!), which together with (3) places severe **constraints** on physics - not all particle can be correlated!

Multiparticle clusters (in momentum space)



The average number of correlated pairs within a cluster is $\langle r(r-1)/2 \rangle$. Some particles may be unclustered, hence $\langle N_{cl} \rangle \langle r \rangle / \langle n \rangle = \alpha$. Then

$$\sigma_{\text{dyn}}^2 = \frac{\alpha \langle r(r-1) \rangle}{\langle r \rangle \langle n \rangle} \text{cov}^* = \frac{\alpha r^*}{\langle n \rangle} \text{cov}^*, \quad r^* = \frac{\langle r(r-1) \rangle}{\langle r \rangle},$$

which complies to the scaling of σ_{dyn} if $\alpha r^* \text{cov}^*$ is independent of $\langle n \rangle$ (in the fiducial centrality range). For a fixed number of particles in each cluster we have $r^* = \langle r \rangle - 1$, for the Poisson distribution $r^* = \langle r \rangle$, while for wider distributions $r^* > \langle r \rangle$.

Cluster scaling

$$\sum_{i \neq j} \text{cov}(p_i, p_j) \sim \langle n \rangle$$

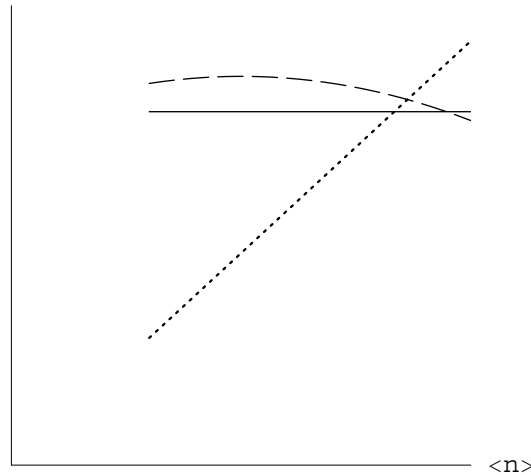
$$\sigma_{\text{dyn}}^2 \sim \frac{1}{\langle n \rangle}$$

$$F_{p_T} \sim 1$$

$$\Phi_{p_T} \sim 1$$

$$\Sigma_{p_T} \sim \frac{1}{\sqrt{\langle n \rangle}}$$

$$\text{STAR } \langle \Delta p_i \Delta p_j \rangle \sim \frac{1}{\langle n \rangle}$$



If the multiplicity of produced particles $\langle n \rangle$ is used - sensitive to final state,
if $\langle n \rangle \rightarrow N_p$ - sensitivity to initial state

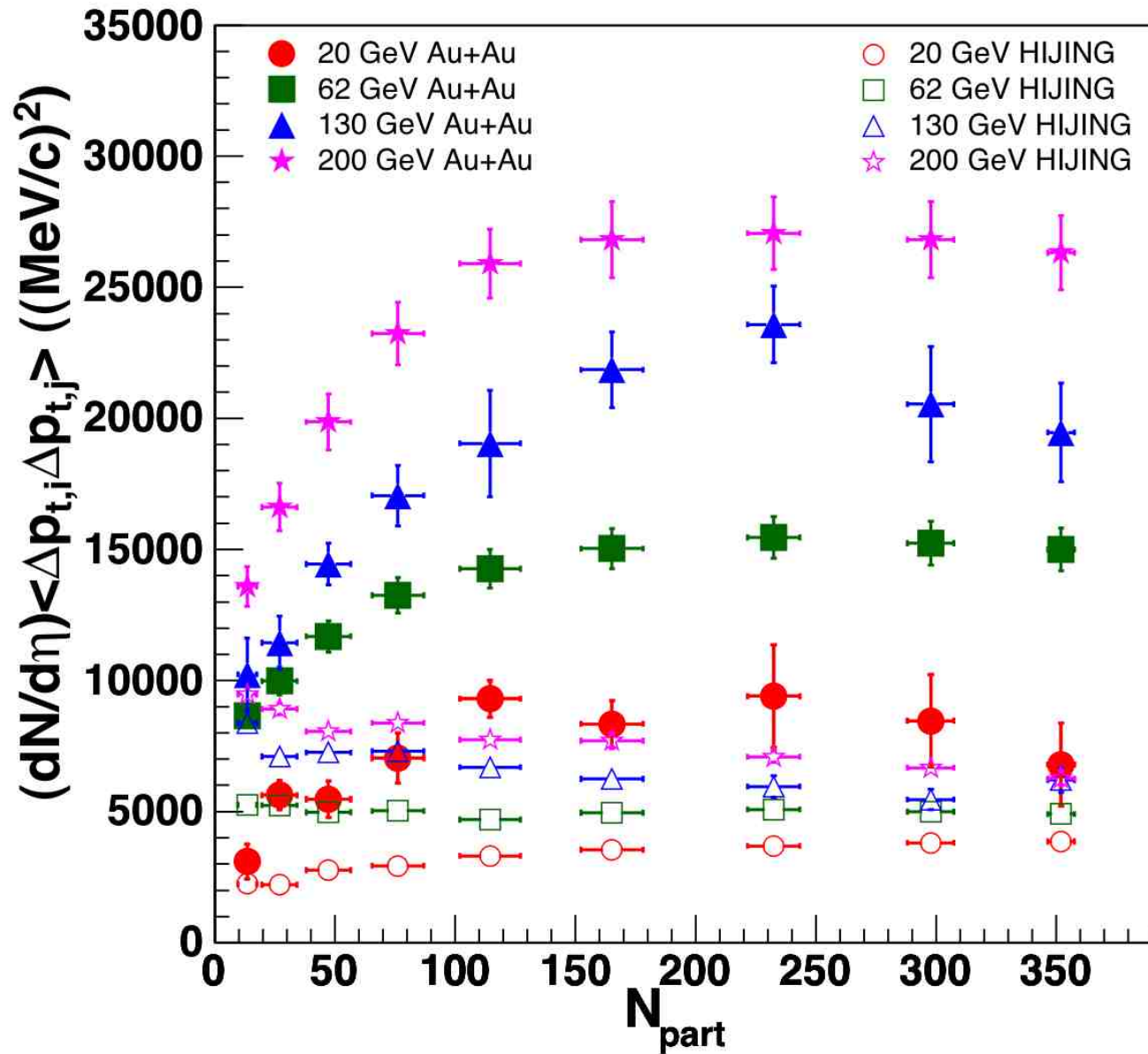
Try both, *i.e* use also just $\langle n \rangle$ to define the classes

Comply to scaling: PHENIX@130, STAR perhaps except for 130, CERES
(within error bars), PHENIX@200 - fixed plane,

Scaling violations: PHENIX@200 (random), NA49 (try $\langle n \rangle$ instead of N_p)

[Paul Sorensen's plot]

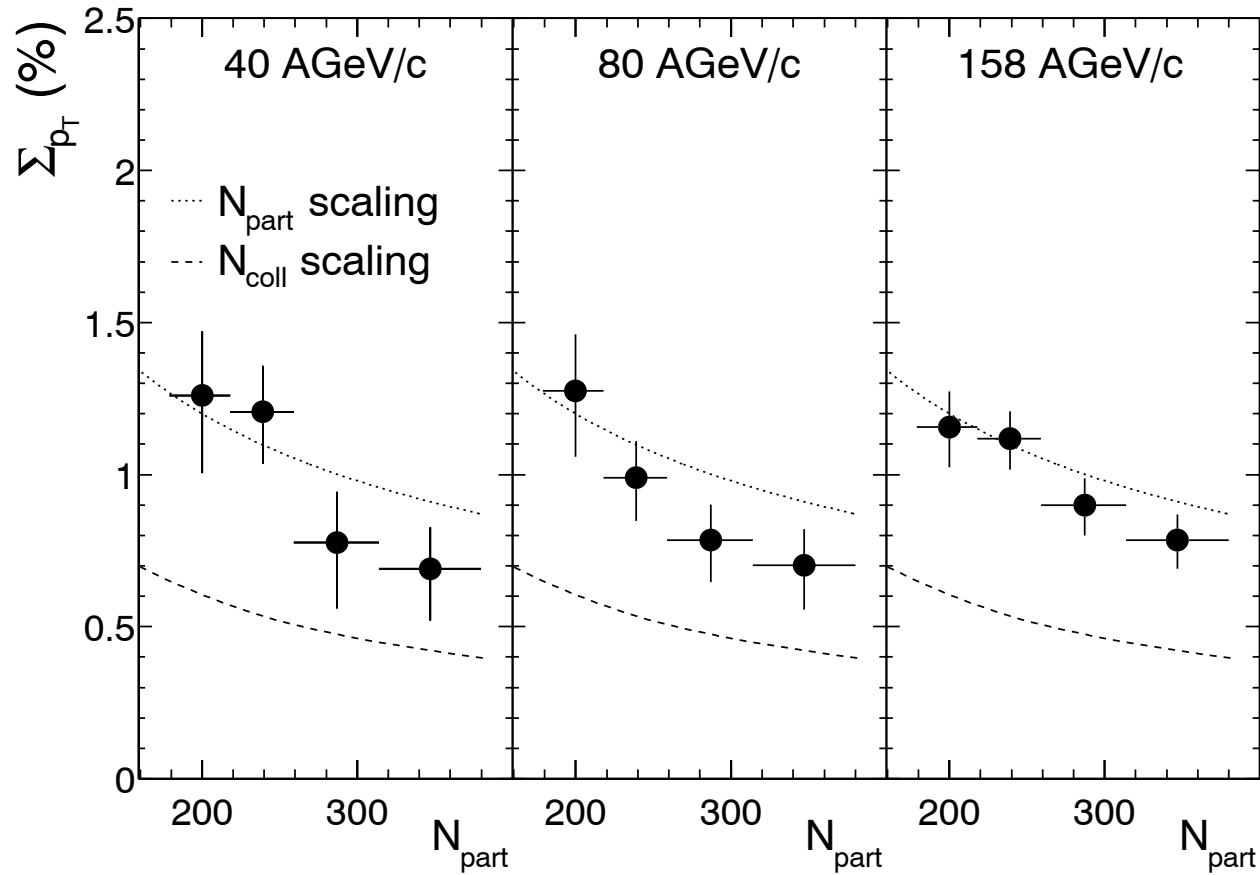
STAR



STAR, hep-ph/0504031

$$\frac{dN}{d\eta}\langle\Delta p_i\Delta p_j\rangle \simeq \sigma_{\text{dyn}}^2\langle n\rangle/\Delta\eta \sim \text{const.}$$

CERES



$$\Sigma_{p_T} \equiv \frac{\sigma_{dyn}}{\langle p_T \rangle} \sim \frac{1}{\sqrt{\langle n \rangle}}$$

Works approximately quite well! (errors large)

How strong are the correlations?

a - detector efficiency, number of observed particles $\sim a$, number of pairs $\sim a^2$. Thus

$$\text{cov}^* = \sigma_{\text{dyn}}^2 \frac{\langle n \rangle}{ar^*}.$$

For PHENIX@130 $a \simeq 10\%$, which gives

$$\text{cov}^* \simeq \frac{0.035 \text{ GeV}^2}{r^*}.$$

The natural scale set by $\sigma_p^2 \simeq 0.08 \text{ GeV}^2$ (recall that $|\text{cov}^*| \leq \sigma_p^2$). For $r = 2$ the value of cov^* would assume 45% of the maximum possible value. This is unlikely, as argued from model estimates presented below, where cov^* at most 0.01 GeV^2 . Thus a natural explanation of the above number is to take a **significantly larger value of r^*** . The higher r^* , the easier it is to satisfy the data even with small values of cov^* .

Same for STAR

Very similar quantitative conclusions from the STAR data [nucl-ex/0504031]. The measure used by STAR is the estimator for σ_{dyn}^2 :

$$\langle \Delta p_i \Delta p_j \rangle = \frac{N_{\text{event}} - 1}{N_{\text{event}}} \sigma_M^2 - \frac{1}{N_{\text{event}}} \sum_{k=1}^{N_{\text{event}}} \frac{\sigma_p^2}{n_k} \simeq \sigma_{\text{dyn}}^2 \quad (1)$$

Assuming $a = 0.75$ we find

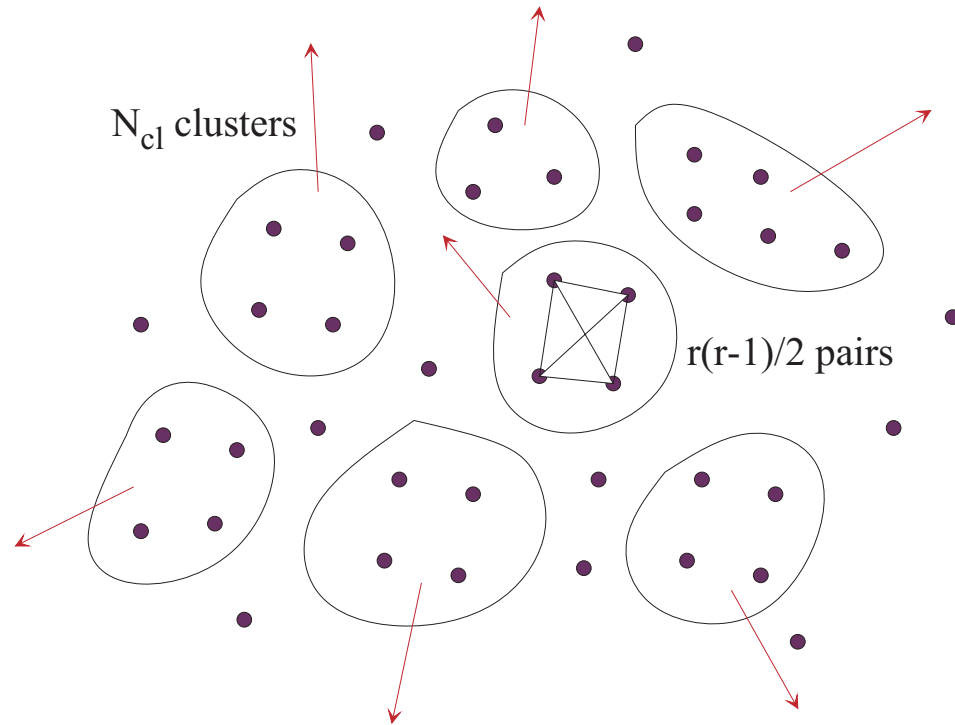
$$\text{cov}^* r^* = 0.058, 0.043, 0.035, 0.014 \text{ GeV}^2$$

for $\sqrt{s_{NN}} = 200, \quad 130, \quad 62, \quad 20 \text{ GeV}$

The value at 130 GeV is close to PHENIX. Significant beam-energy dependence! This may be due to **increase of the covariance per pair with energy, and/or increase of the number of clustered particles**

What is the nature of clusters?

- - (mini)jets, resonances, droplets of matter receding at the same collective velocity



“Lumped clusters”: lumps of matter move at some collective velocities, correlating the momenta of particles belonging to the same cluster

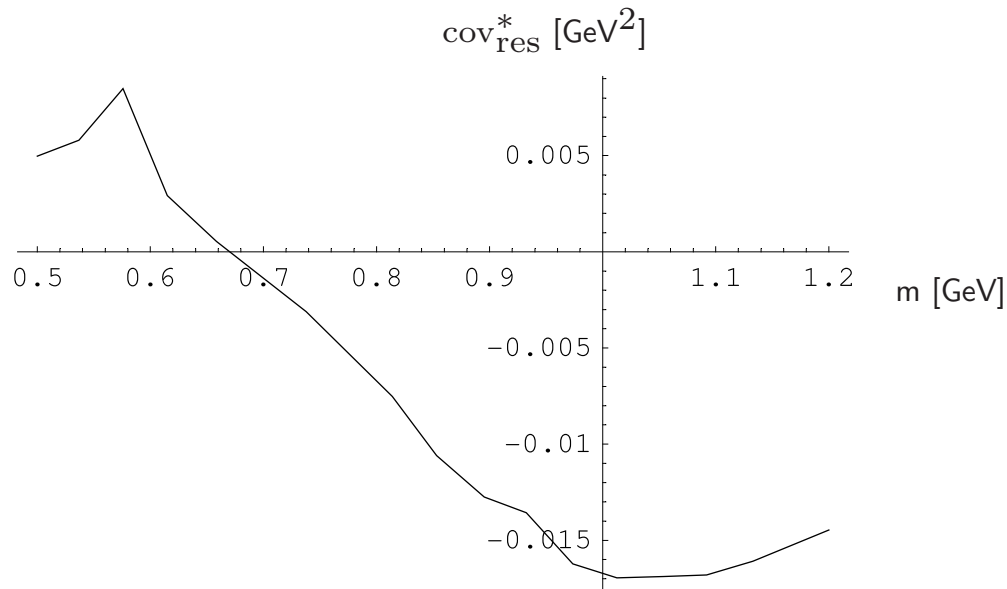
Covariance from the decay of resonances

$$\text{cov}_{\text{res}}^* = \frac{\int d^3p \int \frac{d^3p_1}{E_{p_1}} \int \frac{d^3p_2}{E_{p_2}} \delta^{(4)}(p - p_1 - p_2) C \frac{dN_R}{d^3p} (p_1^\perp - \langle p^\perp \rangle) (p_2^\perp - \langle p^\perp \rangle)}{\int d^3p \int \frac{d^3p_1}{E_{p_1}} \int \frac{d^3p_2}{E_{p_2}} \delta^{(4)}(p - p_1 - p_2) C \frac{dN_R}{d^3p}}$$

dN_R/d^3p - resonance distribution from the Cooper-Frye formula with the [single freezeout model](#), p_1, p_2 - momenta of daughters, E_p - energy of the particle with momentum p , C - cuts

chocov2.nb

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Cancellations between contributions of various resonances are possible; Terminator - negligible contribution of resonances to the p_T correlations. Accidental zero near the ρ -meson mass, need of accurate implementation of cuts.

Thermal clusters

Emission from local thermalized sources: each element of the fluid moves with its collective velocity and emits particles with locally thermalized spectra. The picture refelects charge conservation within the local source [Bożek, WB, Florkowski, Acta Phys. Hung. A22 (2005) 149].

$$\text{cov}_{i,j}^* = \frac{\int d\Sigma_\mu u^\mu \int d^3p_1 (p_1^\perp - \langle p^\perp \rangle) f_i^u(p_1) \int d^3p_2 (p_2^\perp - \langle p^\perp \rangle) f_j^u(p_2)}{\int d\Sigma_\mu u^\mu \int d^3p_1 f_i^u(p_1) \int d^3p_2 f_j^u(p_2)}$$

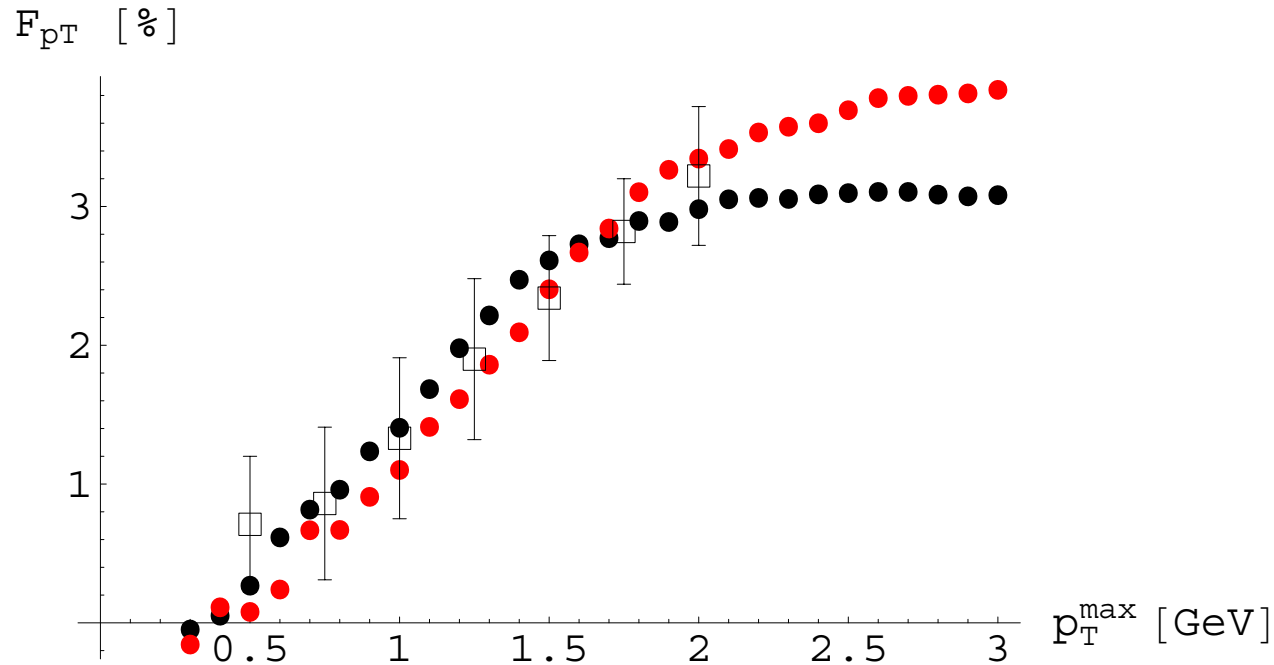
$f_i^u(p) = (\exp(p \cdot u/T) \pm 1)^{-1}$ - boosted thermal distribution, $u(x)$ -expansion velocity, $d\Sigma_\mu$ - integration over the freeze-out hypersurface. Fix flow such that $\langle M \rangle = 554$ MeV

T [MeV]	10	100	120	140	165	200
$\langle \beta \rangle$	0.94	0.72	0.69	0.58	0.49	0.31
σ_p^2 [GeV ²]	0.056	0.19	0.15	0.15	0.14	0.12
cov^* [GeV ²]	0.027	0.011	0.0088	0.0063	0.0034	0.0006

Results depend strongly on temperature. At realistic thermal parameters the experimental value of the covariance, $0.035 \text{ GeV}^2/r^*$, cannot be accounted for unless the number of (charged) particles belonging to a cluster is sizeable, **at least 5 – 10**

$p_{\perp}^{\max}$ dependence: data - PHENIX Au+Au @ 200, $c = 20 - 25\%$

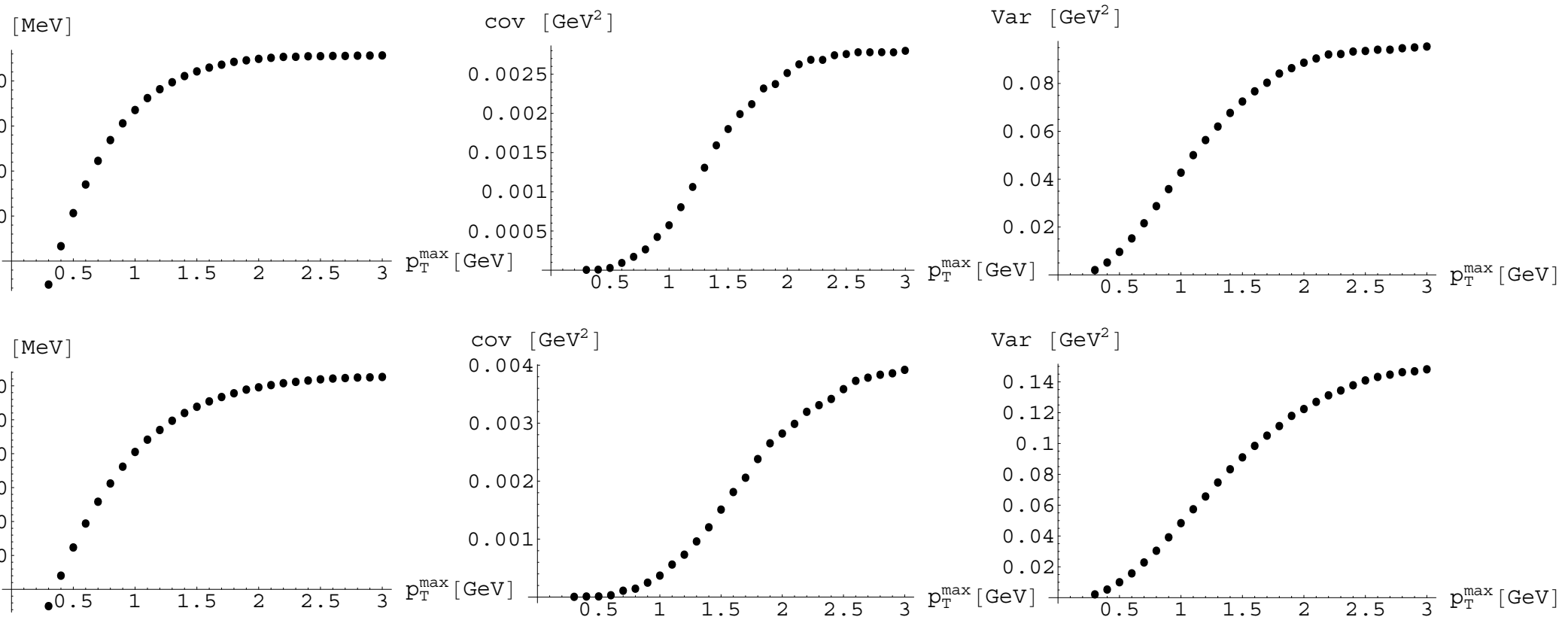
thermal clusters - red: $T = 165$ MeV, $ar^* = 2.1$, black: $T = 130$ MeV
(T lowered due to resonance decays !!!), $ar^* = 2.9$



$$\omega = \frac{\sigma_M}{\langle M \rangle}, \quad F_{pT} = \frac{\omega_{\text{data}} - \omega_{\text{mix}}}{\omega_{\text{mix}}} \simeq \frac{\text{COV}^* ar^*}{2\sigma_p^2}$$

Why soft physics goes to $p_T \sim 2$ GeV

top: $T = 130$ MeV, bottom: $T = 165$ MeV



Conclusion

1. A “combinatoric” attempt
2. Constant $\langle M \rangle$ and σ_p in the fiducial centrality range made the calculation easy, otherwise somewhat more involved formulas are needed but the analysis is straightforward. Need more experimental info.
3. Appearance of **scaling of σ_{dyn}^2 with $1/\langle n \rangle$** (and appropriately in other *equivalent* measures of correlations) suggest the **cluster picture** of the fireball
4. This **cluster scaling** can be also seen at STAR and at CERES
5. Use also $\langle n \rangle$, not N_p only.
6. The clusters may a priori originate from very different physics: (mini)jets, droplets of fluid formed in the explosive scenario of the collision, or other mechanisms leading to multiparticle correlations
7. The magnitude of the observed σ_{dyn} can be easier achieved when several (4-10 charged) particles are present in a cluster (thermal clusters estimate)
8. In the thermal clusters model F_{p_T} grows linearly with $p_{\perp}^{\text{max}}$ and then saturates around 2 GeV
9. More detailed microscopic modelling necessary