# Nucleon structure from lattice QCD

M. Göckeler, P. Hägler, R. Horsley, Y. Nakamura, D. Pleiter, P.E.L. Rakow, A. Schäfer, G. Schierholz, W. Schroers, H. Stüben, Th. Streuer, J.M. Zanotti

QCDSF Collaboration

*apeNEXT: Computational Challenges and First Physics Results,*
GGI, Firenze

08. February 2007

# Outline

# Content

# Introduction

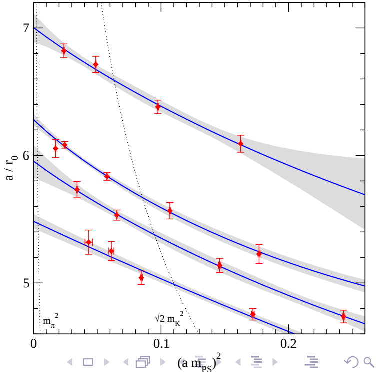
- Lattice in principle allows for ab initio calculation of
  - Moments of nucleon structure functions
  - Nucleon form factors (electromagnetic FF, GFF)
- Good control of several **systematic errors** related to extrapolations needed:
  - **Infinite volume limit** $V \to \infty$
  - **Continuum limit** $a \to 0$
  - **Chiral limit** $m_{\mathrm{q}} \to 0$
- Calculations now feasible for 2-3 flavours of dynamical fermions thanks to
  - Improved **algorithms** (e.g. mass preconditioning, multiple timescales)
  - New **capability computers** (e.g. BG/L, apeNEXT)
- This talk: selected results from QCDSF collaboration on nucleon structure

# Simulation parameters

Configurations with $N_f = 2$ O(a)-improved dynamical quarks generated by UKQCD+QCDSF+DIK.

$$m_{\mathrm{PS,sea}} = 340, \ldots, 1170 \ \mathrm{MeV} \qquad a = 0.07, \ldots, 0.11 \ \mathrm{fm}$$
$$m_{\mathrm{PS,val}} = 340, \ldots, 1240 \ \mathrm{MeV} \qquad V = 1.4, \ldots, 2.6 \ \mathrm{fm}$$

- Simulations much closer to the physical quark mass
- Reasonably small lattice spacing
- Volumes at lower limit



# Momenta and polarisations

- 3 initial state momentum:

$$\frac{L}{2\pi}\vec{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- 3 choices for polarisations:

$$\begin{aligned} \Gamma &= (1+\gamma_4)/2 \\ \Gamma &= (1+\gamma_4)\mathrm{i}\gamma_5\gamma_1/2 \\ \Gamma &= (1+\gamma_4)\mathrm{i}\gamma_5\gamma_2/2 \end{aligned}$$

- 17 different choices of momentum transfer $\vec{q} = \vec{p}\,' - \vec{p}$
- Configurations at distance 5-10 trajectories, 16-4 different sources
- Use binning to eliminate auto-correlations

# Calculation of matrix elements

- Matrix elements are determined from ratios:

$$R(t,\tau,\vec{p}\,',\vec{p}\,) = \frac{C_3(t,\tau,\vec{p}\,',\vec{p}\,)}{C_2(t,\vec{p}\,')} \times \left[\frac{C_2(\tau,\vec{p}\,')C_2(t,\vec{p}\,')C_2(t-\tau,\vec{p}\,)}{C_2(\tau,\vec{p}\,)C_2(t,\vec{p}\,)C_2(t-\tau,\vec{p}\,')}\right]^{1/2}$$

where

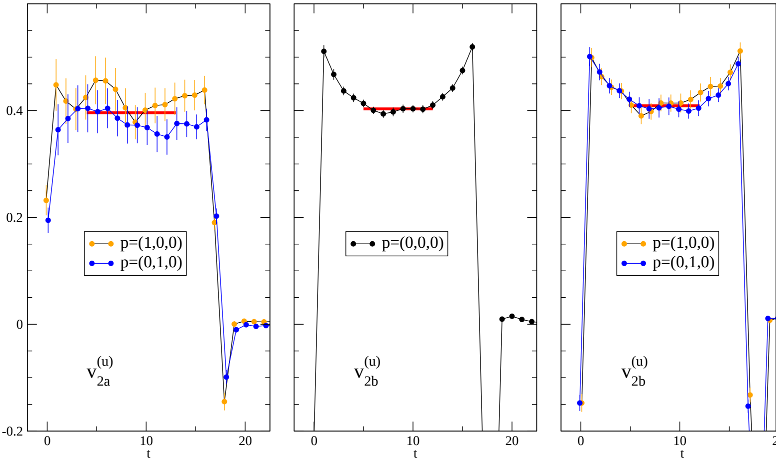
$$C_2(t,\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

and

$$C_3(t,\tau,\vec{p}\,',\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,'\,)\mathcal{O}(\tau)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

- Non-perturbative renormalisation using, e.g., Rome-Southampton method (RI′-MOM scheme)

# Example: $v_{2a}$ vs. $v_{2b}$




$(\beta, \kappa_{\rm sea}) = (5.4, 0.13560)$, V=$24^3 \times 48$

# Content

# Introduction

- Form factor of the proton axial current:

$$\langle p', s' | A_\mu^{u-d} | p, s \rangle = \bar{u}(p', s') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2m_N} G_P(q^2) \right] u(p, s)$$

- Axial coupling constant: $g_A = G_A(0)$
- Parton model interpretation: Fraction of the nucleon spin carried by the quarks $\Delta q$

$$\langle p, s | A_\mu^{u-d} | p, s \rangle = 2 g_A s_\mu = 2(\Delta u - \Delta d) s_\mu$$

- Precise experimental value for $g_A$

# Chiral effective field theory

[QCDSF 2006]

- Evaluation of matrix element of isovector axial current within a low energy effective theory of QCD feasible:

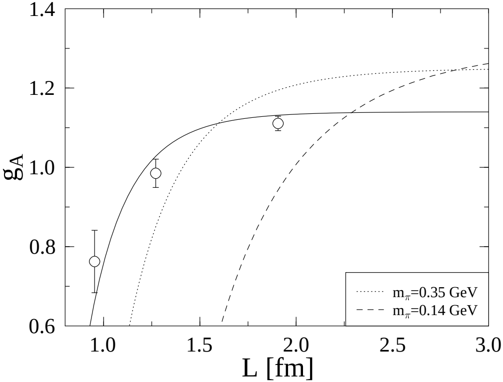
$$g_A(m_{\mathrm{PS}})$$

- Calculation for finite spatial cubic box of length $L$:

$$g_A(m_{\mathrm{PS}}, L) = g_A(m_{\mathrm{PS}}) + \Delta g_A(m_{\mathrm{PS}}, L)$$

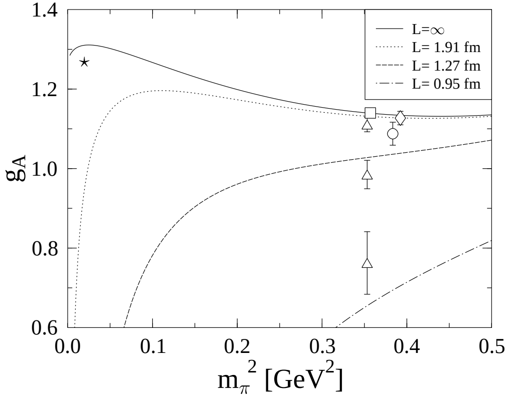
- Lattice results not sufficient to fix all parameters $\rightarrow$ phenomenological input required:
  - Pion decay constant
  - $N\Delta$ mass splitting $\Delta_0$ and axial coupling $c_A$
  - Coupling $B_{20}^r(\lambda)$

# Exploring finite size effects




- Results for $g_A$ at $\beta = 5.29$, $\kappa = 0.1359 \to m_{\mathrm{PS}} \simeq 0.6\ \mathrm{GeV}$

# Chiral extrapolation




- Consistent description of lattice and experimental results
- Results for small quark masses ($m_{PS}^2 < 0.1\ \mathrm{GeV}^2$) and large volumes ($L > 2\ \mathrm{fm}$) needed

# Content

# Introduction

$$\int_0^1 dx\, x^{n-2}\, F^{\mathrm{u-d}}(x, Q^2) = f\, E^{\mathcal{S}}_{F;v_n}\left(\frac{M^2}{Q^2}, g^{\mathcal{S}}(M)\right) v_n{}^{\mathcal{S}}(g^{\mathcal{S}}(M))$$

- Matrix elements $\mathbf{v_n}$ can be measured on the lattice

$$\langle N(\vec{p})| \left[\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} - \mathrm{Tr}\right] |N(\vec{p})\rangle^{\mathcal{S}} := 2v_n^{(q)\mathcal{S}}\ [p^{\mu_1}\cdots p^{\mu_n} - \mathrm{Tr}]$$

- Results have to be renormalised using a scheme (e.g. $\overline{\mathrm{MS}}$) and scale (e.g. 2 GeV).

# Chiral extrapolation

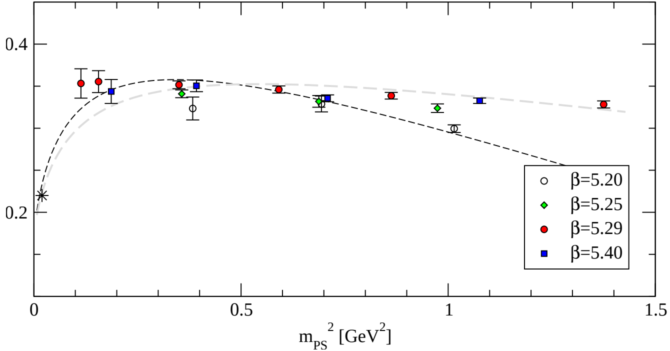
Fit function model taking 'pion cloud' around nucleon into account:

[Thomas et al., Detmold et al.]

$$
\begin{aligned}
v_n^{\mathrm{RGI}}(r_0 m_{\mathrm{PS}}) &= F^{v_n}(r_0 m_{\mathrm{PS}}) \\
F^{v_n}(x) &= v_n^{\mathrm{RGI}} \left( 1 - C x^2 \ln \frac{x^2}{x^2 + (r_0 \Lambda_\chi)^2} \right) + a_n x^2
\end{aligned}
$$

# Lattice results for $v_{2b}^{\mathrm{RGI}}$

- Fit to $\beta = 5.29$ data, $\Lambda_\chi = 500\ \mathrm{MeV}$, experimental value fixed




- Fit ansatz does not describe data very well
- No indication found for strong FSE

# Content

# Introduction

- Standard decomposition of nucleon electromagnetic matrix elements:

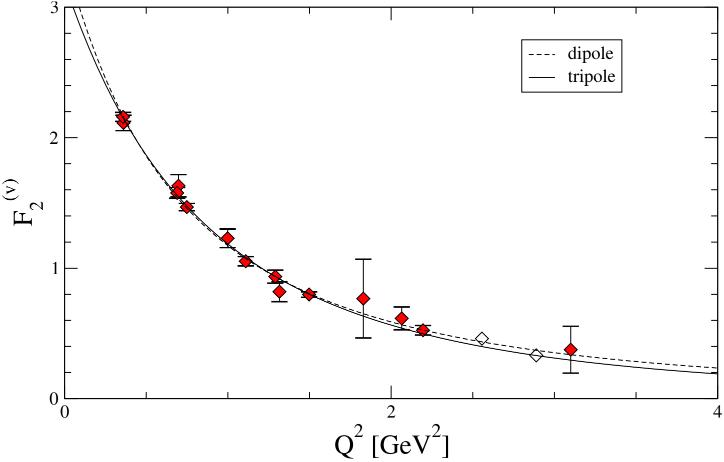
$$\langle p', s' | J^\mu | p, s\rangle = \overline{u}(p', s') \left[\gamma_\mu \mathrm{F_1(q^2)} + i\sigma^{\mu\nu}\frac{q_\nu}{2M_N}\mathrm{F_2(q^2)}\right] u(p, s)$$

- We will consider, e.g.

| | |
|---|---|
| Proton form factors: | $\frac{2}{3}\overline{u}\gamma^\mu u - \frac{1}{3}\overline{d}\gamma^\mu d$ |
| Isovector form factors: | $\overline{u}\gamma^\mu u - \overline{d}\gamma^\mu d$ <br> $\rightarrow$ Disconnected terms cancel |

- Experimental interest:
  - **$q^2$ scaling**
  - **Flavour dependence**

# Typical result




- Difference of fits small wrt to statistical errors

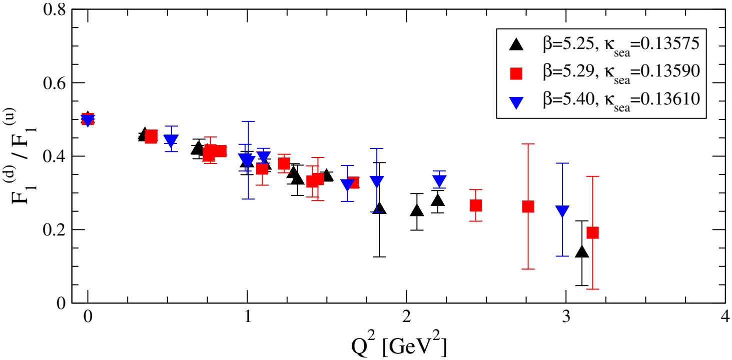
# Parametrisation of lattice results

- Fit data to

$$F_i(q^2) = \frac{A_i}{(1 - q^2/M_i^2)^p}$$

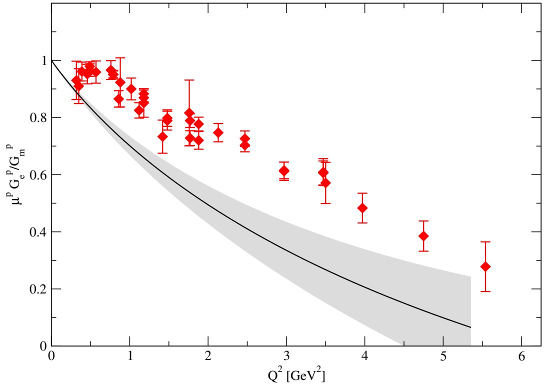
- Fit ansatz determines
  - $F_1^{(q)}(0)$, $F_2^{(q)}(0)$ ($q = u, d$)
  - Di-/tripole masses or, equivalently, the form factor radii
- Lattice data not precise enough to fix $p$
- Looking for minimal $\chi^2$ suggests:
  - $p = 2$ for $F_1^{(u)}$
  - $p = 3$ for $F_1^{(d)}$, $F_2^{(u)}$, $F_2^{(d)}$

# Scaling of $F_1^{(d)}/F_1^{(u)}$



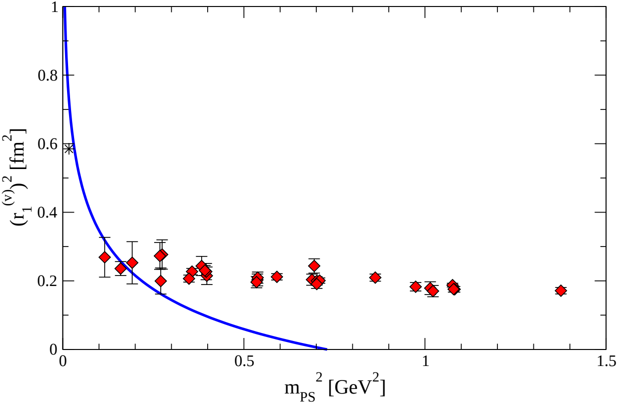

- Data suggests $p = 2$ and $p = 3$ to fit $F_1^{(u)}$ and $F_1^{(d)}$, respectively
- Flavour dependence also observed in fits to experimental data [Diehl et al., 2005]

# Scaling of $\mu^{(p)} G_e^{(p)}(q^2)/G_m^{(p)}(q^2)$




- Ignore disconnected terms
- Naive extrapolation (linear in quark mass)
- **But:** Naive extrapolation is not correct

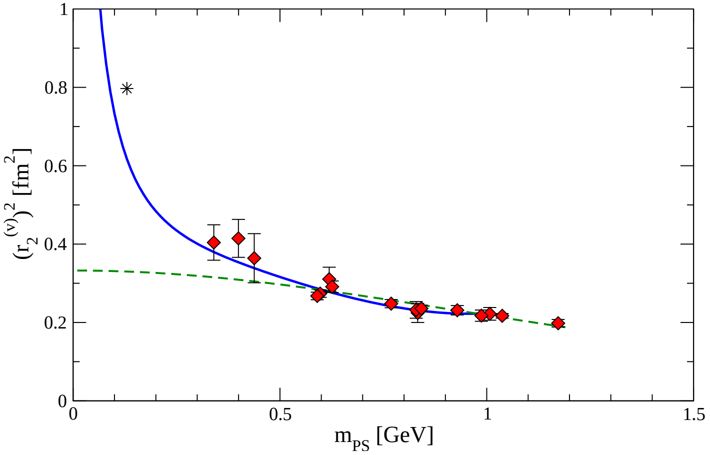
# Dirac radius $[r_1^{(v)}]^2$



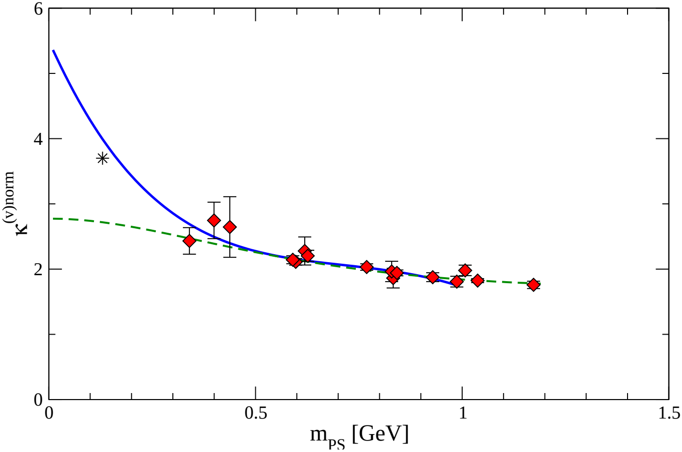

- Calculation of isovector radii within a low energy effective theory of QCD predicts strong quark mass dependence

# Pauli radius $[r_2^{(v)}]^2$

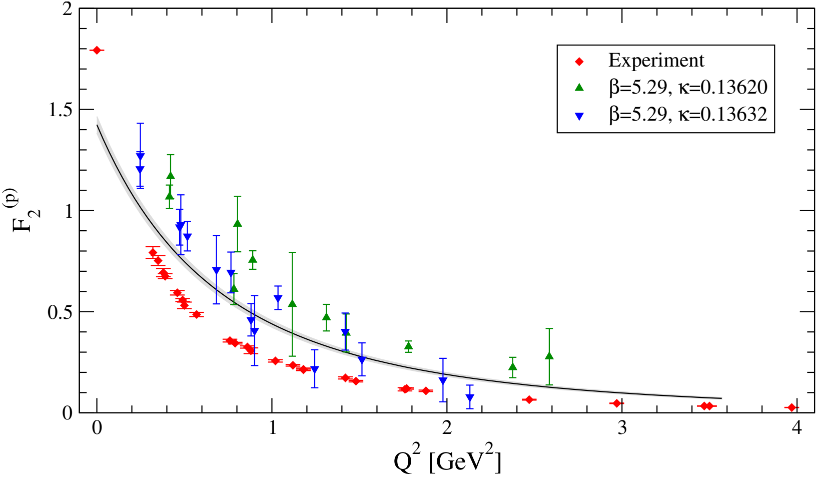
- Joined fit to $[r_2^{(v)}]^2$ and $\kappa^{(v)}$:



# Anomalous magnetic moment $\kappa^{(v)}$



# $F_2^{(p)}$: Lattice vs. Experiment




- Better agreement for $m_{\mathrm{PS}} \simeq 300$ MeV?

# Content

# Introduction

- Lattice methods allow for calculation of moments of the quark density $\rho(x, b_\perp, s_\perp, S_\perp)$

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) =$$

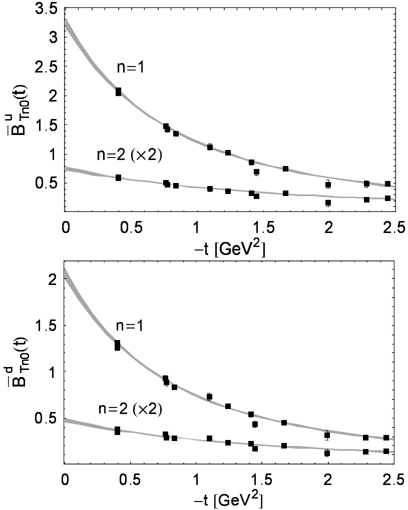
$$\frac{1}{2}\Big\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \widetilde{A}_{Tn0}(b_\perp^2) \right)$$

$$+ \frac{b_\perp^j \epsilon^{ji}}{m} \left( S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2) \right)$$

$$+ s_\perp^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \widetilde{A}''_{Tn0}(b_\perp^2) \Big\}$$

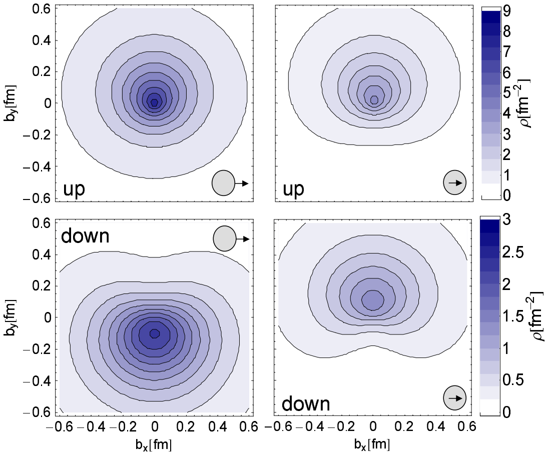
  - momentum fraction $x$
  - transverse spin $s_\perp$
  - distance $b_\perp$ from the center-of-momentum
  - transverse spin $S_\perp$

# Generalized form factor $\overline{B}_{T(n=1,2)0}(t)$



# Quark densities in nucleon

- Parametrize lattice data using p-pole fits
- Fourier transformation to impact parameter $b_\perp$ space



# Content

# APE and the QCDSF physics program

## APE machines at NIC/DESY (Zeuthen)

| | |
|---|---|
| APE100 | 1994-2004 |
| APEmille | 2000-.... |
| apeNEXT | 2005-.... |

Selected bibliography:

- 1994: Towards a Lattice Calculation of the Nucleon Structure Functions
- 1997: Pion and Rho Structure Functions from Lattice QCD
- 1999: A Lattice Determination of Light Quark Masses
- 2000: A lattice calculation of the nucleon's spin-dependent structure function $g_2$ revisited
- 2001: Determination of $\Lambda_{\overline{MS}}$ from quenched and $N_f = 2$ dynamical QCD
- 2002: A lattice study of the spin structure of the Lambda hyperon
- 2003: Nucleon electromagnetic form factors on the lattice and in chiral effective field theory
- 2004: A lattice determination of moments of unpolarised nucleon structure functions using improved Wilson fermions
- 2005: Quark helicity flip generalized parton distributions from two-flavor lattice QCD
- 2006: Moments of pseudoscalar meson distribution amplitudes from the lattice

QCDSF program was only possible with APE

# International Lattice Datagrid

- Huge efforts required to push towards $m_q \to 0$, $V \to \infty$, $a \to 0$
  - Collaborations start to become larger
  - Different sources for compute power used

☞ Important to improve on data sharing

- A number of relevant standards have been defined within ILDG
- Started to build-up a (continental) European infrastructure which starts to become heavily used:

| | |
|---|---|
| SESAM/T$\chi$L/GRAL | O(50,000) |
| ETMC | O(25,000) |
| QCDSF | O(20,000) |
| total | O(95,000) |

- MWWG goal: Interoperability for download operations by LAT'07

# Conclusions and outlook

- Lattice continues to be interesting method to explore hadron structure:
  - Axial coupling $g_A$, moments of unpolarised structure functions
  - Electro-magnetic form factors and GFF (transverse spin structure)
- Hadron structure is hot topic for various experimental programs (e.g. JLAB, GSI)
- Control on systematic errors improved but is often not sufficient
  - Possibly large **finite size effects**
  - Strong Quark mass dependence for $m_{\mathrm{PS}} \to m_\pi$
    ☞ **chiral extrapolations** critical
  - Discretisation effects seem to be relatively small
    ☞ **O(a) improvement program** successful
- Exploring the light quark mass region starts to become feasible
  ☞ **Exciting prospect**
- **But:** Computing power $O(100)$ TFlops (sustained) required