

Electroweak loop corrections at TeV colliders

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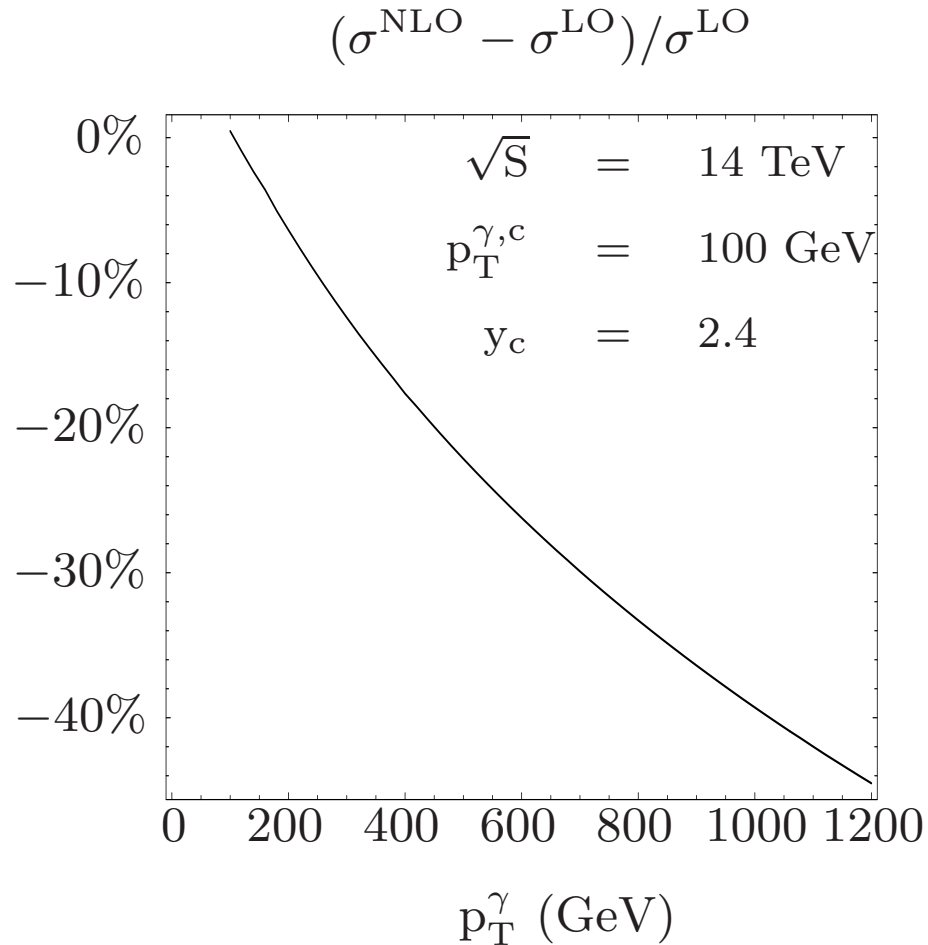
Outline of the talk

- (1) **Electroweak corrections at the TeV scale**
- (2) **One-loop calculations for LHC** [Accomando, Denner, Hollik, Kasprzik, Kniehl, Kaiser, Meier, Kühn, Kulesza, S.P., Schulze, Maina, Moretti, Nolten, Ross, Dittmaier, Krämer, Baur, Wackerot, Zykunov, Beccaria, Mirabella, Marcorini, Carloni Calame, Montagna, Piccinini, Gounaris, Layssac, Renard, Verzegnassi, Ciafaloni, Comelli, Scharf, Uwer]
- (3) **Beyond one loop with resummations** [Fadin, Lipatov, Martin, Melles, Jantzen, Kühn, Moch, Penin, Smirnov, M.Ciafaloni, P.Ciafaloni, Comelli]
- (4) **Diagrammatic two-loop calculations** [Melles, Hori, Kawamura, Kodaira, Beenakker, Werthenbach, Jantzen, Kühn, Moch, Penin, Smirnov, Denner, S.P.]

PART 1

Introduction: electroweak corrections at the TeV scale

Example: electroweak corrections to $pp \rightarrow Z\gamma$ at the LHC



Hollik, Meier (2004)

At small p_T

- Corrections of $\mathcal{O}(\alpha) \sim 1\%$

At $p_T > 100 \text{ GeV}$

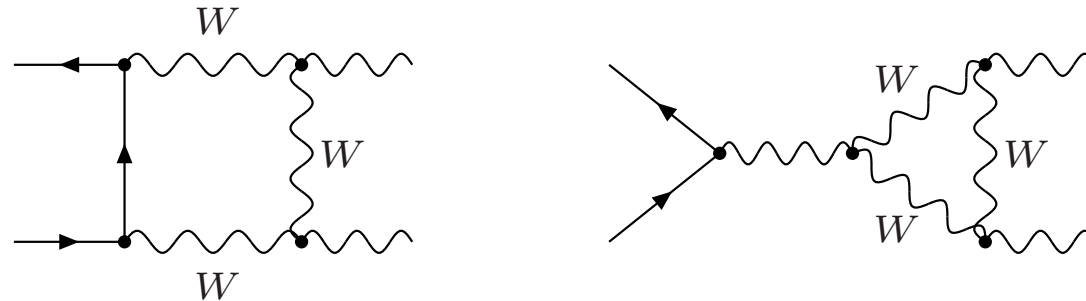
- large negative corrections $\gg 1\%$
- increase with p_T
- -40% at $p_T \sim 1 \text{ TeV}$!

Origin: scattering energy $\gg$ characteristic scale of EW corrections

Large double logarithms

$$\frac{\delta\sigma}{\sigma} \sim -\frac{\alpha}{\pi s_W^2} \ln^2 \left(\frac{s}{M_W^2} \right) \simeq -26\% \quad \text{at} \quad \sqrt{s} \sim 1 \text{ TeV}$$

from vertex and box diagrams involving **virtual weak bosons**



Kuroda, Moutaka, Schildknecht (1991); Deggrasi, Sirlin (1992); Beenakker, Denner, Dittmaier, Mertig, Sack (1993); Denner, Dittmaier, Schuster (1995); Denner, Dittmaier, Hahn (1997), Beccaria, Montagna, Piccinini, Renard, Verzegnassi (1998); Ciafaloni, Comelli (1999)

Affect all hard scattering processes at LHC and ILC!

Asymptotic expansion of 1-loop EW corrections

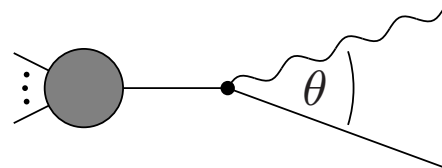
General form of M_W^2/s expansion

$$\propto \left[C_2 \underbrace{\ln^2 \left(\frac{s}{M_W^2} \right)}_{\text{soft, coll}} + C_1 \underbrace{\ln \left(\frac{s}{M_W^2} \right)}_{\text{soft, coll}} + \tilde{C}_1 \underbrace{\ln \left(\frac{s}{\mu_R^2} \right)}_{\text{UV}} + C_0 \right]$$

Terms of $\mathcal{O}(M_W^2/s)$ negligible for $s \sim 1 \text{ TeV}^2$

$$C_k = \sum_{j=0}^{\infty} C_k^{(j)} \left(\frac{M_W^2}{s} \right)^j \rightarrow C_k^{(0)}$$

Mass singularities from **soft/collinear gauge bosons** coupling to external lines



$$\Rightarrow \int \frac{dE}{E} \int \frac{d \cos \theta}{(1 - \cos \theta)}$$

Analogies with massless gauge theories (QED,QCD)

Key to understand electroweak logarithms

$$M^2/s \ll 1 \quad \Rightarrow \quad \ln\left(\frac{s}{M^2}\right) \quad \text{mass singularities!}$$

Analogous singularities in massless QCD

$$M^2 = 0 \quad \Rightarrow \quad \frac{1}{\varepsilon} \quad \text{in } D = 4 - 2\varepsilon$$

Factorization and Universality of QCD mass singularities [[Kunszt,Soper,Catani](#)]

$$\text{1 loop} \quad \mathcal{M}^{(1)} = \underbrace{I^{(1)}(\varepsilon)}_{1/\varepsilon^2, 1/\varepsilon} \mathcal{M}^{(0)} + \mathcal{O}(1)$$

$$\text{2 loops} \quad \mathcal{M}^{(2)} = \underbrace{I^{(2)}(\varepsilon)}_{1/\varepsilon^4, 1/\varepsilon^3} \mathcal{M}^{(0)} + \mathcal{O}(\varepsilon^{-2})$$

Singular factors $I^{(N)}(\varepsilon)$ process-independent!

Factorization and universality of one-loop EW logarithms [Denner, S.P. (2001)]

For **arbitrary processes** $(e, \nu, u, d, t, b, \gamma, Z, W^\pm, H, g)$

$$\begin{array}{c} \text{1} \quad \text{3} \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \text{2} \quad \text{n} \end{array} = \underbrace{\left(\frac{1}{2} \sum_{j \neq i} \begin{array}{c} i \\ \diagdown \quad \diagup \\ \text{wavy line } W, Z, \gamma \\ \diagup \quad \diagdown \\ j \end{array} \right)}_{\text{universal}} \times \begin{array}{c} \text{1} \quad \text{3} \\ \diagdown \quad \diagup \\ \text{tree} \\ \diagup \quad \diagdown \\ \text{2} \quad \text{n} \end{array}$$

proven with **collinear Ward identities** for spontaneously broken YM theories

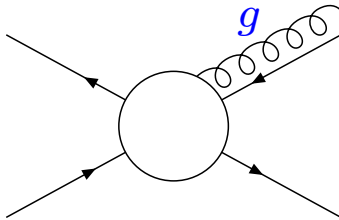
$$\begin{array}{c} i \\ \diagdown \quad \diagup \\ \text{wavy line } W, Z, \gamma \\ \diagup \quad \diagdown \\ j \end{array} = \frac{\alpha}{4\pi} \left\{ \sum_{V=\gamma, Z, W} \mathbf{I}_i^V \mathbf{I}_j^V \ln^2 \frac{r_{ij}}{M_W^2} + 2 \mathbf{I}_i^Z \mathbf{I}_j^Z \ln \frac{r_{ij}}{M_W^2} \ln \frac{M_W^2}{M_Z^2} + \gamma_{ij}^{\text{ew}} \ln \frac{s}{M_W^2} \right. \\ \left. + Q_i Q_j \sum_{k=i,j} \left[\ln \frac{r_{ij}}{m_k^2} \ln \frac{M_W^2}{\lambda^2} - \frac{1}{2} \ln^2 \frac{M_W^2}{m_k^2} - \ln \frac{M_W^2}{\lambda^2} - \frac{1}{2} \ln \frac{M_W^2}{m_k^2} \right] \right\}$$

Simple and general recipe for **LL** and **NLL**

Mass singularities and physical observables

Massless gauge theories

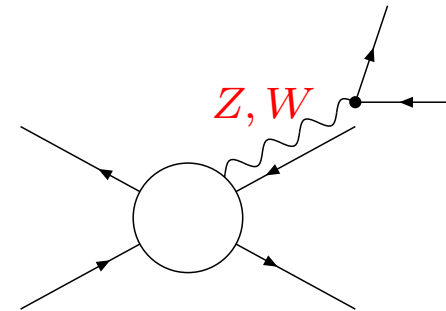
- **real emission** of soft and collinear massless particles **cannot be detected**



- $1/\epsilon$ mass singularities cancel (KLN theorem)

Electroweak theory

- **real emission** of massive W and Z bosons **can be detected**



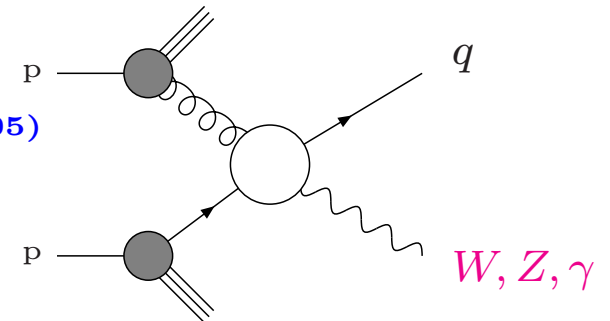
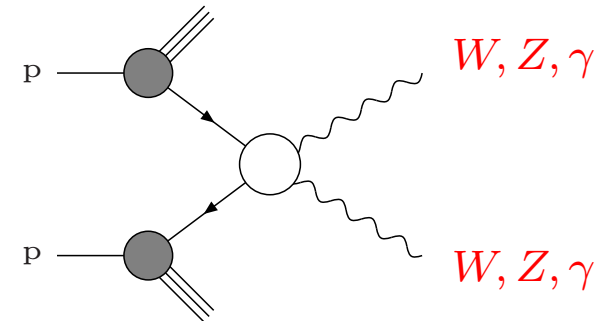
- $\ln\left(\frac{s}{M^2}\right)$ corrections remain present in exclusive observables
- and even in fully inclusive observables
[M. Ciafaloni, P. Ciafaloni, Comelli (2000)]

PART 2

One-loop calculations for LHC

Recent $pp \rightarrow VV$ and $pp \rightarrow Vj$ calculations

process	calculation	
$W\gamma, WZ$	1,3	Accomando, Denner, S.P. (2002)
$Z\gamma$	2,4	Hollik, Meier (2004)
WW, WZ, ZZ	1,2,3	Accomando, Denner, Kaiser (2005)
$W\gamma, Z\gamma$	1,2,3,4	Accomando, Denner, Meier (2005)
$\gamma\gamma$		
Z,γ	4	Maina, Moretti, Ross (2004)
Z,γ	3,4,5	Kühn, Kulesza, S.P., Schulze (2004,2005)
W	2,3,4,5	Kühn, Kulesza, S.P., Schulze (2007)
W	2,4	Hollik, Kasprzik, Kniehl (2007)



¹decay of weak bosons

²real photon bremsstrahlung

³one-loop NLL approximation

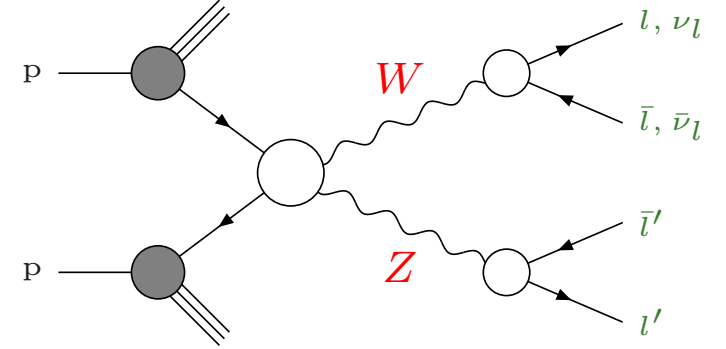
⁴exact one-loop predictions

⁵two-loop NLL approximation

Double-pole and NLL approximations

Factorization and separation of scales

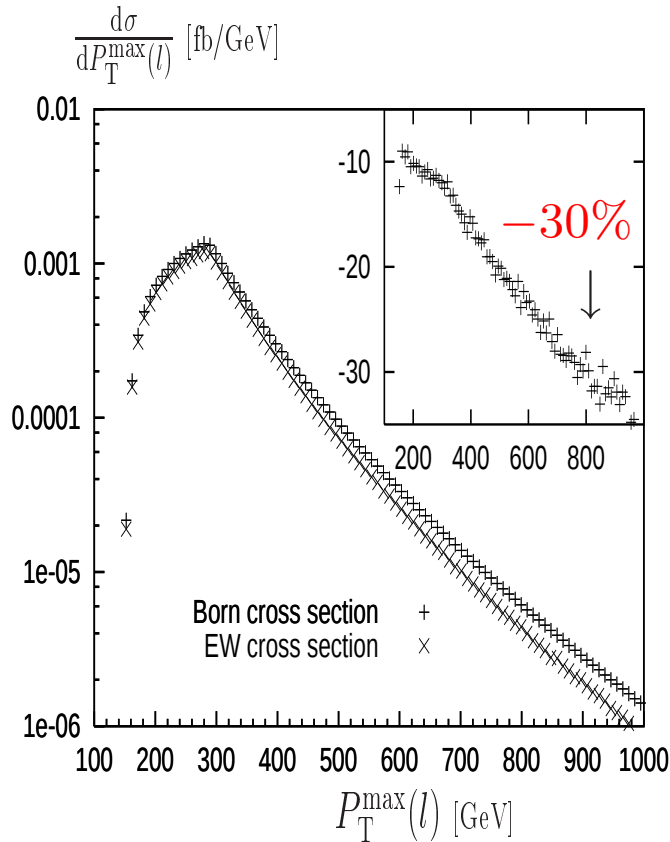
$$\mathcal{M}_{\text{DPA}} = \frac{-\sum_{\lambda,\lambda'} \mathcal{M}^{qq' \rightarrow W_\lambda Z_{\lambda'}} \mathcal{M}^{W_\lambda \rightarrow l\nu_l} \mathcal{M}^{Z_{\lambda'} \rightarrow l'\bar{l}'}}{(p_W^2 - M_W^2 + iM_W\Gamma_W)(p_Z^2 - M_Z^2 + iM_Z\Gamma_Z)}$$



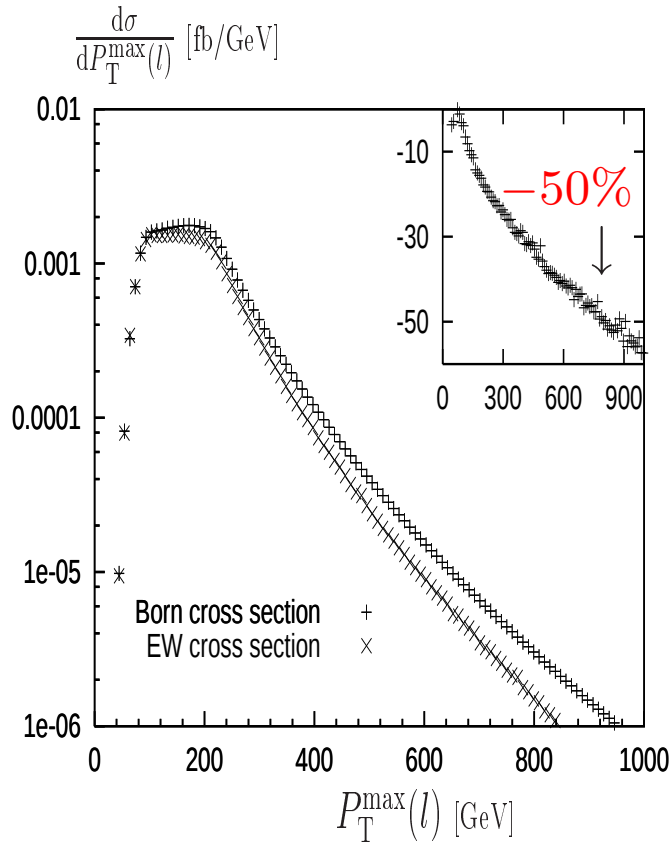
NLL corrections for T/L gauge-boson production

$$\begin{aligned} \mathcal{M}^{\bar{q}q' \rightarrow W_T Z_T} &= \mathcal{M}_{\text{Born}}^{\bar{q}q' \rightarrow W_T Z_T} \left\{ 1 - \frac{\alpha}{8\pi} \left\{ 2C_{q_L}^{\text{ew}} + C_W^{\text{ew}} \left[1 + \frac{c_W^2 \cos \hat{\theta}}{c_W^2 \cos \hat{\theta} - s_W^2 Y_{q_L}} \right] \right\} \ln^2 \frac{\hat{s}}{M_W^2} \right. \\ &\quad \left. + \frac{\alpha}{4\pi} \left\{ 3C_{q_L}^{\text{ew}} - \frac{1}{s_W^2} \left[\ln \frac{\hat{t}\hat{u}}{\hat{s}^2} + \frac{c_W^2 \ln(\hat{t}/\hat{u})}{c_W^2 \cos \hat{\theta} - s_W^2 Y_{q_L}} \right] \right\} \ln \frac{\hat{s}}{M_W^2} \right\} \\ \mathcal{M}^{\bar{q}q' \rightarrow W_L Z_L} &= \mathcal{M}_{\text{Born}}^{\bar{q}q' \rightarrow W_L Z_L} \left\{ 1 - \frac{\alpha}{4\pi} [C_{q_L}^{\text{ew}} + C_\Phi^{\text{ew}}] \ln^2 \frac{\hat{s}}{M_W^2} + \frac{\alpha}{4\pi} \left[-\frac{2}{s_W^2} \left(\ln \frac{|\hat{u}|}{\hat{s}} \right. \right. \right. \\ &\quad \left. \left. + \ln \frac{|\hat{t}|}{\hat{s}} - \frac{s_W^2}{c_W^2} Y_{q_L} \ln \frac{\hat{t}}{\hat{u}} \right) 3C_{q_L}^{\text{ew}} + 4C_\Phi^{\text{ew}} - \frac{3}{2s_W^2} \frac{m_t^2}{M_W^2} - b_2^{(1)} \right] \ln \frac{\hat{s}}{M_W^2} \right\} \end{aligned}$$

Size of electroweak NLL corrections at the LHC



$$pp \rightarrow \text{WZ} \rightarrow e\nu_e \mu^+ \mu^-$$



$$pp \rightarrow \text{ZZ} \rightarrow e^+ e^- \mu^+ \mu^-$$

High- p_T region

- corrections $\gg 10\%$!
- size process-dependent

Importance

- small cross section
- important for new-physics searches

Accomando, Denner, Kaiser (2005)

Precision of NLL/NNLL approx. for $pp \rightarrow W\gamma \rightarrow l\nu_l\gamma$ [Accomando, Denner, Meier (2005)]

Complete high-energy expansion vs exact one-loop calculation

$$\underbrace{C_2 \ln^2 \left(\frac{\hat{s}}{M_W^2} \right) + C_1 \ln \left(\frac{\hat{s}}{M_W^2} \right)}_{\text{NLL}} + \underbrace{B_0 + B_1 \ln \left(\frac{\hat{t}}{\hat{s}} \right) + B_2 \ln^2 \left(\frac{\hat{t}}{\hat{s}} \right)}_{\text{NNLL (not enhanced)}} + \underbrace{\mathcal{O} \left(\frac{M_W^2}{\hat{s}} \right)}_{\text{suppressed}}$$

$p_T(\gamma)/\text{GeV}$	NLL	const	$\ln(\hat{t}/\hat{s})$	$\ln^2(\hat{t}/\hat{s})$
250	-6.05%	-0.26%	-1.78%	-3.95%
450	-14.4%	-0.58%	-1.97%	-3.50%
700	-22.6%	-0.74%	-1.91%	-3.05%
1000	-30.1%	-0.87%	-1.79%	-2.62%

suppr: numerically $< 0.5\%$ $\Rightarrow$ asymptotic high-energy expansion good approach

NLL: Dominant (20-30%). Agreement with Denner, S.P. (2001).

Correct description of energy dependence.

NNLL.: Less important (-5.5%) but larger than expected ($\alpha \sim 1\%$) and not negligible.
(Analytic expression? Other processes?)

Precision of NLL/NNLL approx. for $pp \rightarrow Zj$ [Kühn, Kulesza, S.P., Schulze (2005)]

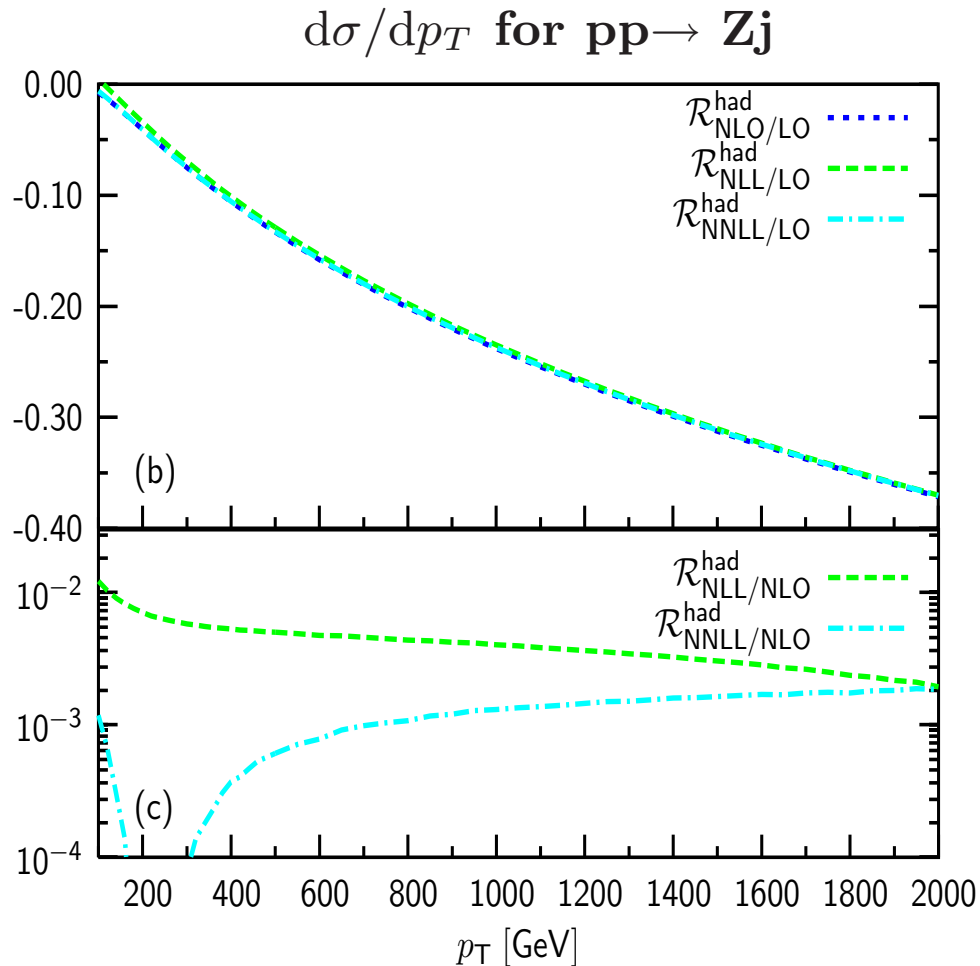
Asymptotic expansion ($|\hat{s}|, |\hat{t}|, |\hat{u}| \gg M_W^2$) for $q\bar{q} \rightarrow Zg$ amplitude

$$\begin{aligned}
 |\overline{\mathcal{M}}_1|^2 = & 32\pi^2 \alpha^2 \alpha_S \sum_{\lambda=R,L} \left\{ \left(I_{q\lambda}^Z \right)^2 \sum_{V=Z,W^\pm} \left(I^V I^{\bar{V}} \right)_{q\lambda} \left\{ \left[-\ln^2 \left(\frac{-\hat{s}}{M_V^2} \right) + 3 \ln \left(\frac{-\hat{s}}{M_V^2} \right) + \frac{3}{2} [\ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) \right. \right. \right. \\
 & + \ln^2 \left(\frac{\hat{u}}{\hat{s}} \right) + \ln \left(\frac{\hat{t}}{\hat{s}} \right) + \ln \left(\frac{\hat{u}}{\hat{s}} \right) \left. \right] + \frac{7\pi^2}{3} - \frac{5}{2} \left] \left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} \right) + \frac{1}{2} \left[3 \ln \left(\frac{\hat{u}}{\hat{s}} \right) - 3 \ln \left(\frac{\hat{t}}{\hat{s}} \right) - \ln^2 \left(\frac{\hat{u}}{\hat{s}} \right) + \ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) \right] \right. \\
 & \times \left(\frac{\hat{t}^2 - \hat{u}^2}{\hat{t}\hat{u}} \right) + 2 \left[\ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) + \ln^2 \left(\frac{\hat{u}}{\hat{s}} \right) + \ln \left(\frac{\hat{t}}{\hat{s}} \right) + \ln \left(\frac{\hat{u}}{\hat{s}} \right) + 2\pi^2 \right] \left. \right\} + \frac{c_W}{s_W^3} T_{q\lambda}^3 I_{q\lambda}^Z \left\{ \left[\frac{4}{4-D} - 2\gamma_E \right. \right. \\
 & + 2 \ln \left(\frac{4\pi\mu^2}{M_Z^2} \right) + \ln^2 \left(\frac{-\hat{s}}{M_W^2} \right) - \ln^2 \left(\frac{-\hat{t}}{M_W^2} \right) - \ln^2 \left(\frac{-\hat{u}}{M_W^2} \right) + \ln^2 \left(\frac{\hat{t}}{\hat{u}} \right) - \frac{3}{2} [\ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) + \ln^2 \left(\frac{\hat{u}}{\hat{s}} \right)] - \frac{20\pi^2}{9} - \frac{\pi}{\sqrt{3}} \\
 & \left. \left. + 2 \left[\left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} \right) + \frac{1}{2} \left[\ln^2 \left(\frac{\hat{u}}{\hat{s}} \right) - \ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) \right] \left(\frac{\hat{t}^2 - \hat{u}^2}{\hat{t}\hat{u}} \right) - 2 \left[\ln^2 \left(\frac{\hat{t}}{\hat{s}} \right) + \ln^2 \left(\frac{\hat{u}}{\hat{s}} \right) + \ln \left(\frac{\hat{t}}{\hat{s}} \right) + \ln \left(\frac{\hat{u}}{\hat{s}} \right) + 2\pi^2 \right] \right] \right\} \right\}
 \end{aligned}$$

Very compact expressions

- **NLL** predicted by process-independent formula [Denner, S.P. (2001)]
- **NNLL** consist of $\pi^2, \pi/\sqrt{3}, \ln(\hat{t}/\hat{u}), \dots$ not growing with energy

NLL/NNLL approximations vs exact calculation



Large negative corrections

- -25% at $p_T \sim 1$ TeV
- **NLO**, **NLL**, **NNLL** overlap!

Precision of **NNLL** approximation

- better than 0.2%

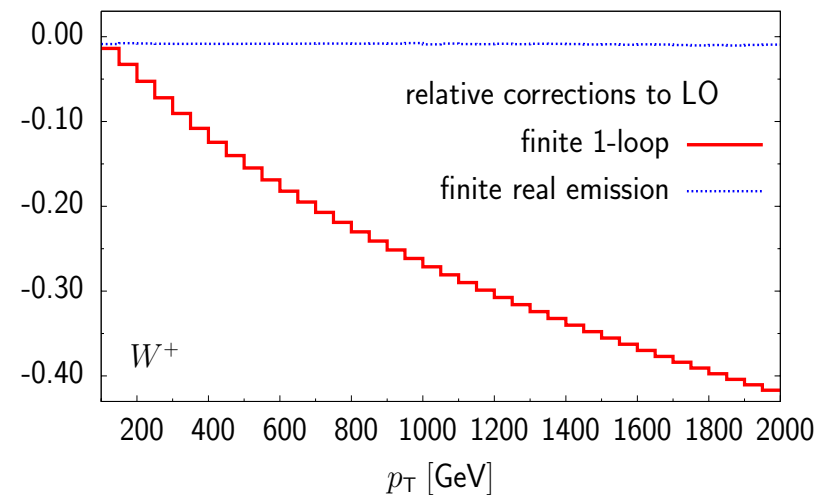
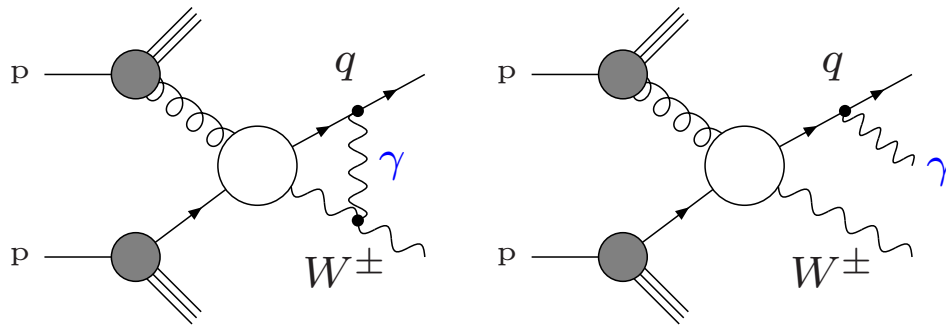
Precision of **NLL** approximation

- better than 1% ! (process-dependent)

$\Rightarrow$ use asymptotic expansions for two-loop EW corrections at high energies

Separation of photonic singularities for $pp \rightarrow Wj$ [Kühn, Kulesza, S.P., Schulze (2007)]

Cancellation of virtual-photon divergencies requires real bremsstrahlung. Needed techniques (dipole subtraction) not available beyond one loop.



Strategy: gauge-invariant splitting

- $\sigma_{\text{virt}}^{\text{fin}} = \sigma_{\text{virt}}(M_\gamma = M_W)$
- $\sigma_\gamma^{\text{fin}}$ = virtual-photon singularities + photon bremsstrahlung

One-loop calculation for $pp \rightarrow Wj$

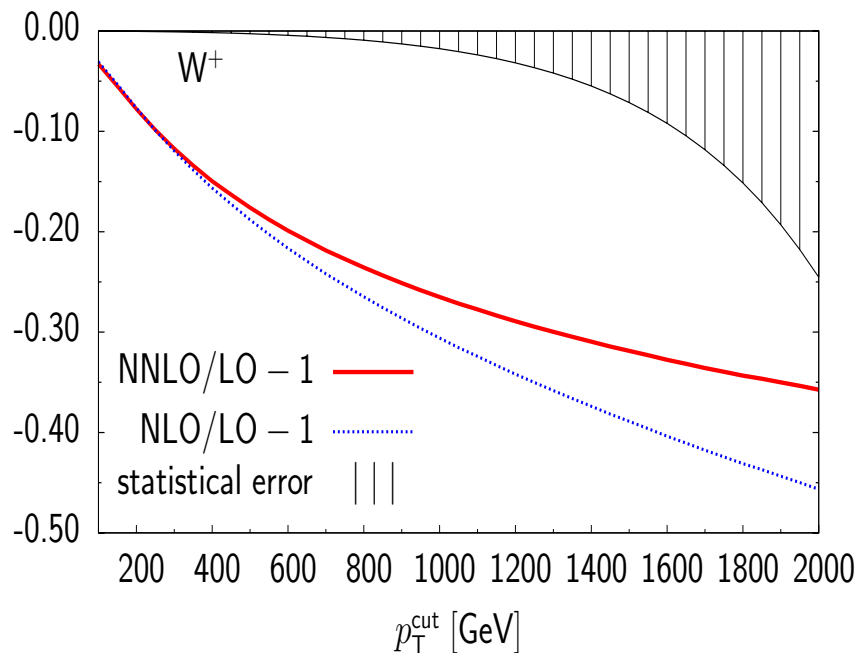
- $\sigma_{\text{virt}}^{\text{fin}}$ = large negative corrections
- $\sigma_\gamma^{\text{fin}} \leq 1\%$ for fully inclusive γ

Two-loop LL and NLL corrections for $pp \rightarrow Wj$ [Kühn, Kulesza, S.P., Schulze (2007)]

Compact formula for partonic scattering amplitudes ($\bar{q}q' \rightarrow Wg$)

$$\begin{aligned} \overline{\sum} |\mathcal{M}_2|^2 = & 4 \frac{\alpha^3 \alpha_S}{s_W^2} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} \left\{ \left(C_{qL}^{\text{ew}} + \frac{C_A}{2s_W^2} \right) \left[\frac{C_A}{2s_W^2} \left(\ln^4 \left(\frac{\hat{t}}{M_W^2} \right) + \ln^4 \left(\frac{\hat{u}}{M_W^2} \right) - \ln^4 \left(\frac{\hat{s}}{M_W^2} \right) \right) \right. \right. \\ & \left. \left. + C_{qL}^{\text{ew}} \left(\ln^4 \left(\frac{\hat{s}}{M_W^2} \right) - 6 \ln^3 \left(\frac{\hat{s}}{M_W^2} \right) \right) \right] + \frac{1}{3} \left[\frac{b_1}{c_W^2} \left(\frac{Y_{qL}}{2} \right)^2 + \frac{b_2}{s_W^2} \left(C_F + \frac{C_A}{2} \right) \right] \ln^3 \left(\frac{\hat{s}}{M_W^2} \right) \right\} \end{aligned}$$

Derived from Melles (2001); Denner, Melles, S.P. (2003) (see later)



Size of **one-loop** and **two-loop** corrections

- $-27\% + 3\% = -24\%$ at $p_T \sim 1$ TeV
- $-42\% + 9\% = -33\%$ at $p_T \sim 2$ TeV

Two-loop $\simeq$ statistical error!

Percent-level precision requires also two-loop EW effects!

- Complete two-loop (analytical or numerical) calculation out of reach
- Asymptotic high-energy expansion for $M_W^2/s \ll 1$

$$\alpha^2 \left[C_4 \underbrace{\ln^4 \left(\frac{s}{M_W^2} \right)}_{\text{LL}} + C_3 \underbrace{\ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{NLL}} + \dots \right]$$

organize calculation according to (formal) hierarchy

$$\ln^4 \left(\frac{s}{M_W^2} \right) \gg \ln^3 \left(\frac{s}{M_W^2} \right) \gg \dots$$

PART 3

Beyond one loop: QCD-inspired resummations

InfraRed Evolution Equation (IREE) for QCD matrix elements

Logarithmic dependence on soft-collinear cut-off $\frac{\partial \mathcal{M}}{\partial \ln(\mu_T)} = K(\mu_T) \mathcal{M}$

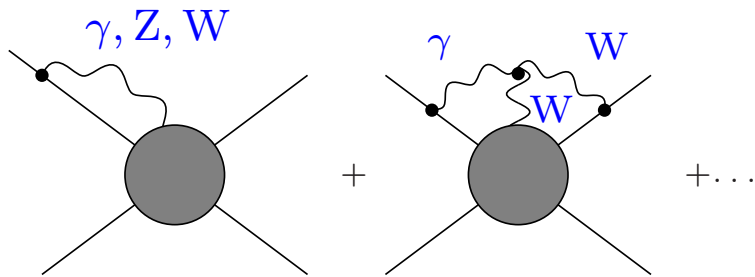
$$\mathcal{M}(\mu_T) = \underbrace{\begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \vdots \end{array}}_{\mathbf{k_T}(\text{gluon}) > \mu_T} = \exp \left[- \int_{\mu_T}^Q \frac{d\mu}{\mu} K(\mu) \right] \mathcal{M}(Q)$$

Gribov(1967); Kirschner, Lipatov (1982)

How to deal with mass gap in the electroweak gauge sector?

$$M_\gamma = 0 \ll M_Z \sim M_W : \quad \begin{array}{c} \text{diagram} \end{array} \Rightarrow \alpha^2 \frac{1}{\epsilon} \ln^3 \left(\frac{s}{M_W^2} \right)$$

$SU(2) \times U(1)$ regime: $\mu_T > M_{W,Z}$

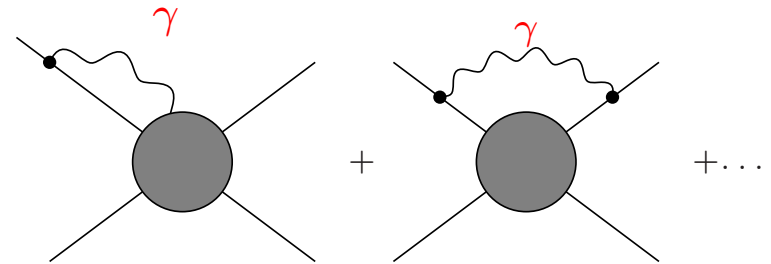


mass gap irrelevant ($M_\gamma = M_Z = M_W$)

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_T)} = K_{EW}(\mu_T) \mathcal{M}$$

as in symmetric $SU(2) \times U(1)$ theory

$U(1)_{em}$ regime: $\mu_T < M_{W,Z}$



weak boson frozen ($M_Z, M_W = \infty$)

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_T)} = K_{QED}(\mu_T) \mathcal{M}$$

as in QED

IREE prediction: double factorization and exponentiation

$$\mathcal{M}(\mu_T) = \exp \left\{ - \int_{\mu_T}^{M_W} \frac{d\mu}{\mu} K_{QED}(\mu) \right\} \exp \left\{ - \int_{M_W}^{\sqrt{s}} \frac{d\mu}{\mu} K_{EW}(\mu) \right\} \mathcal{M}_{\text{Born}}$$

Fadin, Lipatov, Martin, Melles (2000)

Two-loop EW predictions based on IREE approach

Massless $f\bar{f} \rightarrow f'\bar{f}'$ processes

$$\alpha^2 \left[\underbrace{C_4 \ln^4 \left(\frac{s}{M_W^2} \right) + C_3 \ln^3 \left(\frac{s}{M_W^2} \right) + C_2 \ln^2 \left(\frac{s}{M_W^2} \right)}_{\text{Kühn, Moch, Penin, Smirnov (2000,2001,2003)}} + \underbrace{C_1 \ln^1 \left(\frac{s}{M_W^2} \right)}_{\text{Jantzen, Kühn, Penin, Smirnov (2004,2005)}} \right]$$

Main ingredient: 2-loop log corrections for fermionic vertex [Jantzen, Smirnov (2006)]

$$\mathcal{F} = \text{circle with two external lines} = \text{triangle loop with wavy line} + \text{triangle loop with wavy line and external wavy line} + \text{triangle loop with wavy line and bubble loop} + \dots$$

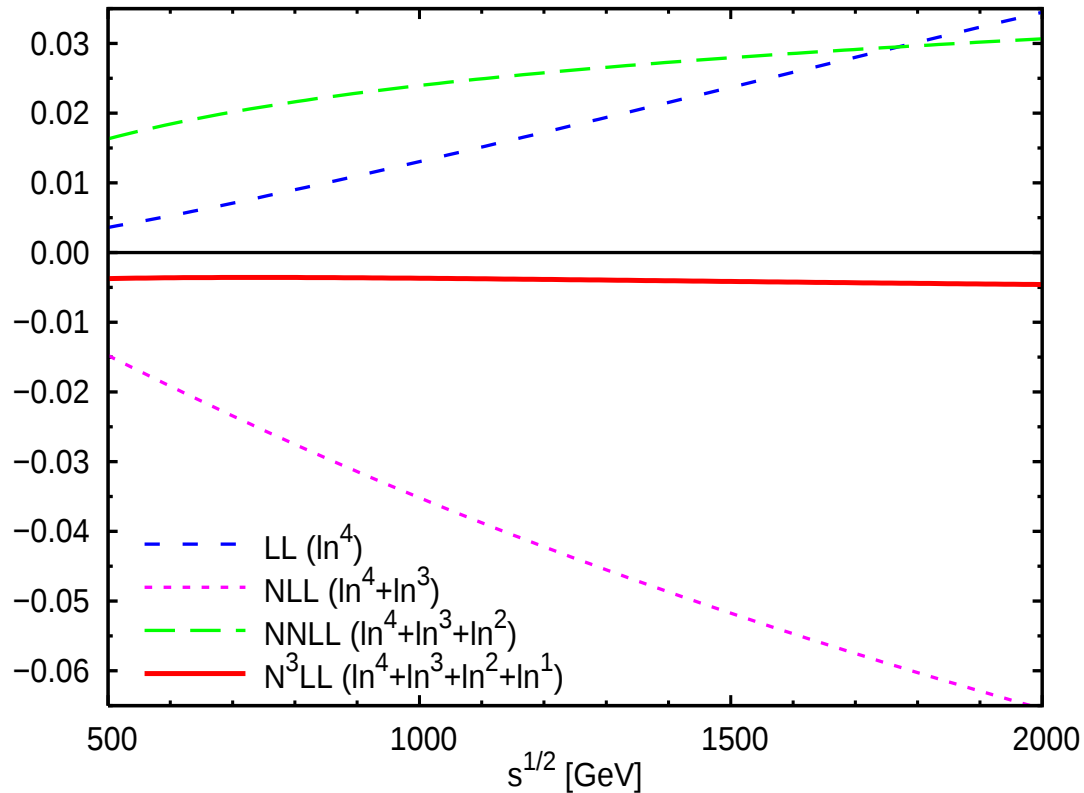
(A) $SU(2) \times U(1)$ Higgs model, $M_\gamma = M_Z = M_W = M_H$, $\sin(\theta_W) = 0 \Rightarrow \ln(s/M_W^2)$

(B) $M_\gamma = 0 \Rightarrow$ check IREE (separation of photonic singularities)

(C)
$$\text{circle with four external lines} = \frac{ig^2}{s} \mathcal{F}^2 \times \text{four-fermion vertex with wavy line loop} \quad (\text{reduced amp. from 2-loop QCD results})$$

Relative 2-loop logarithmic corrections to $\sigma(e^+e^- \rightarrow d\bar{d})$

$$\left(\frac{\alpha}{4\pi \sin^2 \theta_w}\right)^2 \left[2.79 \ln^4 \left(\frac{s}{M_W^2} \right) - 51.98 \ln^3 \left(\frac{s}{M_W^2} \right) + 321.34 \ln^2 \left(\frac{s}{M_W^2} \right) - 757.35 \ln^1 \left(\frac{s}{M_W^2} \right) \right]$$



Subleading logarithms

- increasingly large coefficients
- alternating signs

Importance of logarithmic effects

- total 2-loop correction small
- + residual theoretical error $\mathcal{O}(10^{-3})$

Behaviour of log expansion

- oscillating, bad convergence
- + better convergence expected for gauge-boson production

Other two-loop EW predictions based on IREE approach

Arbitrary processes

$$\alpha^2 \left[\underbrace{C_4 \ln^4 \left(\frac{s}{M_W^2} \right)}_{\text{Fadin, Lipatov, Martin, Melles (2000)}} + \underbrace{C_3 \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{Melles (2001,2002,2003,2004)}} \right]$$

Assumptions

- splitting of EW theory in symmetric $SU(2) \times U(1)$ and QED regime
- symmetry breaking completely negligible

Not proven...

PART 4

Diagrammatic two-loop calculations based on EW Lagrangian

- check IREE-based predictions
- control additional effects: mixing, M_W - M_Z difference

Two-loop calculations based on EW Lagrangian

The (few) existing results agree with the IREE

$$\alpha^2 \left[\underbrace{C_4 \ln^4 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Melles; Hori,Kawamura, Kodaira (2000)} \\ \text{Beenakker, Werthenbach (2000,2002)}}} + \underbrace{C_3^{\text{ang}} \ln \left(\frac{t}{s} \right) \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{Denner, Melles,P. (2003)}} + \underbrace{C_3 \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{S.P. (2004)}} \right]$$

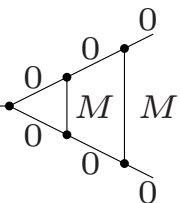
arbitrary processes involving $Z, W, H, b, t, \dots$
massless $f_1 f_2 \rightarrow f_3 \dots f_n$

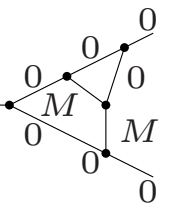
Technology for two-loop NLL for arbitrary processes now available

- automatic algorithm for 2-loop diagrams in NLL approximation
- electroweak collinear Ward identities for process-independent treatment

Algorithm based on sector decomposition [Denner, S.P. (2004)]

- arbitrary two-loop diagrams in the limit $L = \ln(Q^2/M^2) \gg 1$
- photons and light fermions massless in $D = 4 - 2\epsilon$

$$(q_2 p_1)(q_2 p_2) \times \text{Diagram} = - \left(\frac{\mu^2}{Q^2} \right)^\epsilon \left[\frac{1}{16\epsilon} L^2 + \frac{1}{24} L^3 \right]$$


$$(q_2 p_1) \times \text{Diagram} = - \left(\frac{\mu^2}{Q^2} \right)^\epsilon \frac{1}{Q^2} \left[\frac{5}{48} L^4 + \frac{1}{12\epsilon} L^3 + \frac{1-2\gamma_E}{12} L^3 \right]$$


- completely automatized to **NLL accuracy**; computing time = $\mathcal{O}(10\text{s})$

Multi-loop integrals with sector decomposition (one-slide summary)

Hepp(1966); Denner, Roth (1996); Binoth, Heinrich(2000); Denner, S.P. (2004)

(A) **L -loop integral with I propagators: Feynman parametrization**

$$G = \int_0^1 \prod_{i=1}^I d\alpha_j \delta(1 - \sum_{s=1}^I \alpha_s) \frac{\Gamma(e) \mathcal{U}(\vec{\alpha})^{-e}}{[\mathcal{P}(\vec{\alpha}) + (M^2/Q^2) \mathcal{R}(\vec{\alpha})]^f} \quad \text{with } M^2/Q^2 \ll 1$$

(B) **Isolate mass singularities in FP space: sector decomposition**

$$G' = \int_0^1 \prod_{i=1}^m d\beta_i \int_0^1 \prod_{j=1}^n d\alpha_j^f \frac{\mathcal{G}(\vec{\alpha}; \vec{\beta})}{[\alpha_1 \alpha_2 \dots \alpha_n + (M^2/Q^2) \mathcal{H}(\vec{\alpha}; \vec{\beta})]^f} \Rightarrow \ln^n \text{ singularity!}$$

(C) **Extract logarithms: singular α_j -integrations**

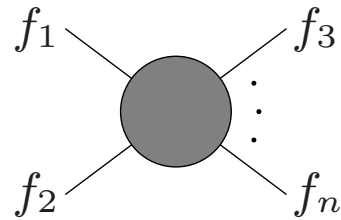
$$G' = \frac{1}{n!} \int_0^1 \prod_{i=1}^m d\beta_i \left\{ \mathcal{G}(\vec{0}, \vec{\beta}) \ln^n \left(\frac{Q^2}{M^2} \right) + (n-1) \left\{ \mathcal{G}(\vec{0}, \vec{\beta}) \left[\ln[\mathcal{H}(\vec{0}, \vec{\beta})] + \sum_{k=1}^{f-1} \frac{1}{k} \right] \right. \right. \\ \left. \left. + \sum_{j=1}^n \int_0^1 \frac{d\alpha_j}{\alpha_j} [\mathcal{G}(0, \dots, \alpha_j, \dots, 0, \vec{\beta}) - \mathcal{G}(\vec{0}, \vec{\beta})] \right\} \ln^{n-1} \left(\frac{Q^2}{M^2} \right) + \mathcal{O}(\ln^{n-2}) \right\}$$

(D) **Compute LL and NLL coefficients: non-singular β_i -integrations**

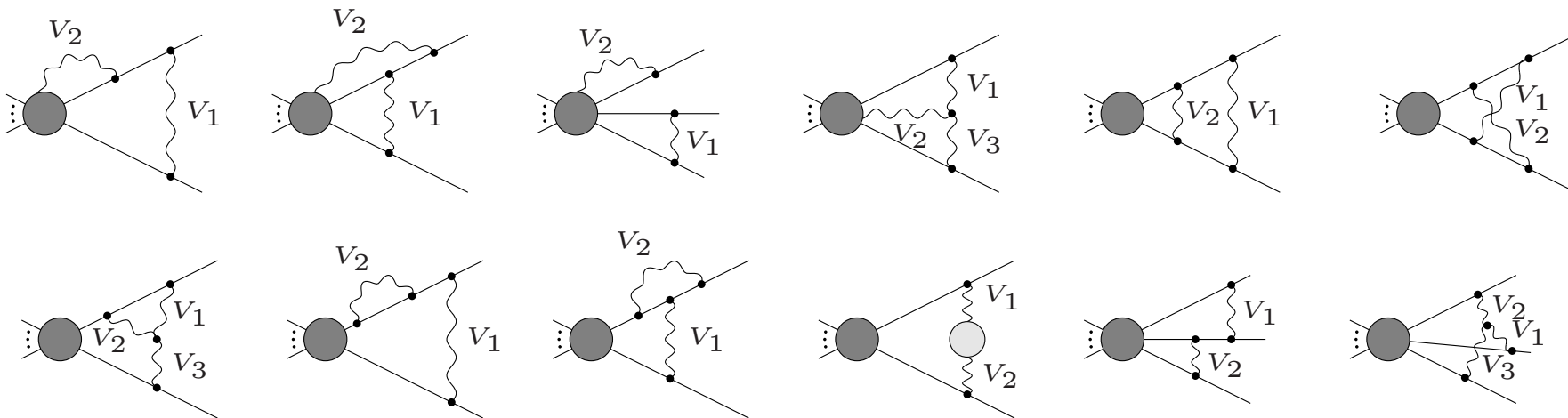
(2-loop diagrams $\Rightarrow$ integrations 2-dimensional and simple)

Computing 2-loop NLL corrections for generic processes

Scattering of n massless fermions [Denner, Jantzen, S.P. (2006)]

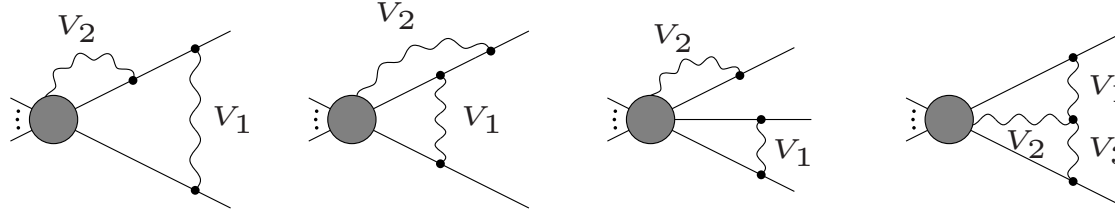


Classification of 2-loop diagrams that yield NLL contributions



(A) Non-factorizable two-loop diagrams

Collinear gauge bosons coupling to external and internal lines



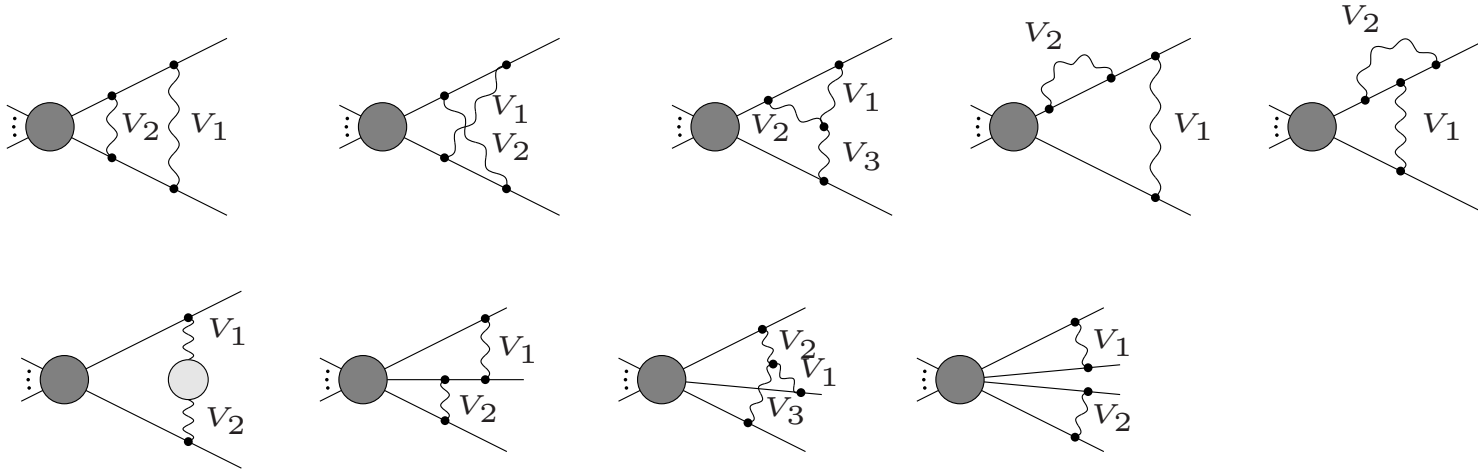
Collinear Ward identities for spontaneously broken non-abelian theories

$$\begin{aligned}
 & \text{Diagram} = \mu_0^{4\epsilon} \int \frac{d^D q_1}{(2\pi)^D} \int \frac{d^D q_2}{(2\pi)^D} \frac{4ie^2 g_2 \epsilon^{V_1 V_2 V_3}}{(q_1^2 - M_{V_1}^2)(q_2^2 - M_{V_2}^2)(q_3^2 - M_{V_3}^2)(p_i - q_2)^2(p_j - q_1)^2} \\
 & \times \lim_{q_1^\mu \rightarrow 0} \lim_{q_2^\mu \rightarrow x p_i^\mu} (p_i - q_2)^{\mu_2} (p_j - q_1)^{\mu_1} \left[g_{\mu_1 \mu_2} (q_1 - q_2)^{\mu_3} + g_{\mu_2}^{\mu_3} (q_2 + q_3)_{\mu_1} - g_{\mu_1}^{\mu_3} (q_3 + q_1)_{\mu_2} \right] \\
 & \times \sum_{\varphi'_i, \varphi'_j} \left\{ G_{\mu_3}^{[\bar{V}_3 \varphi'_i]}(q_3, p_i - q_2) u(p_i, \kappa_i) + \frac{2(p_j + q_2)_{\mu_3}}{(p_j + q_2)^2} \sum_{\varphi''_j} e I_{\varphi''_j \varphi'_j}^{\bar{V}_3} \mathcal{M}_0^{\varphi_1 \dots \varphi'_i \dots \varphi''_j \dots \varphi_n} \right. \\
 & \left. + \sum_{\substack{k=1 \\ k \neq i, j}}^n \frac{2(p_k + q_3)_{\mu_3}}{(p_k + q_3)^2} \sum_{\varphi'_k} \mathcal{M}_0^{\varphi_1 \dots \varphi'_i \dots \varphi'_j \dots \varphi'_k \dots \varphi_n} e I_{\varphi'_k \varphi_k}^{\bar{V}_3} \right\} I_{\varphi'_j \varphi_j}^{\bar{V}_1} I_{\varphi'_i \varphi_i}^{\bar{V}_2} = 0
 \end{aligned}$$

cancellation mechanism permits process-independent treatment

(B) Factorizable two-loop diagrams

Collinear gauge bosons coupling only to external lines



Factorization and explicit calculation using sector decomposition [Denner, S.P. (2004)]
and expansion by regions [Jantzen, Smirnov (2006)]

$$\frac{i e^4 \epsilon^{W \bar{W} \gamma} I_i^W I_i^{\bar{W}} I_j^\gamma}{s_W} \left(\frac{s}{Q^2} \right)^{2\epsilon} \left[-\frac{1}{3} L^3 \epsilon^{-1} - 5 L^4 - 6 \epsilon^{-3} - 6 L \epsilon^{-2} - 2 L^2 \epsilon^{-1} + \frac{2}{3} L^3 \right]$$

Two-loop NLL result for $f_1 f_2 \rightarrow f_3 \dots f_n$

$$\begin{aligned}
 & \text{tree} + \sum_{i,j} \frac{1}{2} \text{diagram} + \sum_{i,j,k,l} \frac{1}{2} \left[\text{diagram} + \text{diagram} \right] + \text{diagram} \\
 & + \text{diagram} + \text{diagram} + \frac{1}{2} \text{diagram} + \text{diagram} + \frac{1}{6} \text{diagram} + \frac{1}{8} \text{diagram} = \\
 & = \exp \left[\sum_{j < i} \text{diagram} \right] \exp \left[\sum_{j < i} \text{diagram} \right] \left[1 + \sum_{j < i} \text{diagram} \right] \text{tree}
 \end{aligned}$$

70 inequivalent two-loop diagrams $\Rightarrow$ very simple result!

- Two-loop $\equiv \exp(1\text{-loop}) \times \text{Born}$
- Agreement with IREE [[Kühn, Moch, Penin, Smirnov \(2000\)](#); [Melles \(2003\)](#)]

Two-loop NLL result for $f_1 f_2 \rightarrow f_3 \dots f_n$

$$\begin{aligned}
 & \text{tree} + \sum_{i,j} \frac{1}{2} \text{diagram} + \sum_{i,j,k,l} \frac{1}{2} \left[\text{diagram} + \text{diagram} \right] + \text{diagram} \\
 & + \text{diagram} + \text{diagram} + \frac{1}{2} \text{diagram} + \text{diagram} + \frac{1}{6} \text{diagram} + \frac{1}{8} \text{diagram} =
 \end{aligned}$$

$$= \exp \left[\sum_{j < i} \text{diagram} \right] \exp \left[\sum_{j < i} \text{diagram} \right] \left[1 + \sum_{j < i} \text{diagram} \right] \text{tree}$$

contains only $L = \ln(s/M_W^2)$ and behaves as in a
symmetric $\text{SU}(2) \times \text{U}(1)$ theory with $M_W = M_Z = M_\gamma$

$$\begin{aligned}
 & \text{diagram} = \left(\frac{\alpha}{4\pi} \right) \sum_{V=B, W^a} I_i^{\bar{V}} I_j^V \left[\underbrace{L^2 + \frac{2}{3} L^3 \epsilon + \frac{1}{4} L^4 \epsilon^2 - \left[\frac{3}{2} - \ln \left(\frac{r_{ij}}{s} \right) \right]}_{K(\epsilon, M_W; r_{ij})} \left(2L + L^2 \epsilon + \frac{1}{3} L^3 \epsilon^2 \right) \right] \\
 & + \left(\frac{\alpha}{4\pi} \right)^2 \frac{1}{2\epsilon} \left[\left(\frac{-s}{\mu^2} \right)^\epsilon K(\epsilon, M_W, s) - K(2\epsilon, M_W, s) \right] \left(g_1^2 \frac{Y_i Y_j}{4} b_1^{(1)} + g_2^2 \frac{T_i^a T_j^a}{4} b_2^{(1)} \right)
 \end{aligned}$$

Two-loop NLL result for $f_1 f_2 \rightarrow f_3 \dots f_n$

$$\begin{aligned}
 & \text{tree} + \sum_{i,j} \frac{1}{2} \text{diagram} + \sum_{i,j,k,l} \frac{1}{2} \left[\text{diagram} + \text{diagram} \right] + \text{diagram} \\
 & + \text{diagram} + \text{diagram} + \frac{1}{2} \text{diagram} + \text{diagram} + \frac{1}{6} \text{diagram} + \frac{1}{8} \text{diagram} =
 \end{aligned}$$

$$= \underbrace{\exp \left[\sum_{j < i} \text{diagram} \right]}_{\text{photonic singularities factorize and behave as in QED}} \exp \left[\sum_{j < i} \text{diagram} \right] \left[1 + \sum_{j < i} \text{diagram} \right] \text{tree}$$

photonic singularities factorize
and behave as in QED

$$\begin{aligned}
 \text{diagram} &= \left(\frac{\alpha}{4\pi} \right) Q_i Q_j \underbrace{\left[2\epsilon^{-2} + 3\epsilon^{-1} - 2\epsilon^{-1} \ln \left(\frac{r_{ij}}{s} \right) - K(\epsilon, M_W; r_{ij}) \right]}_{\Delta K(\epsilon, 0, r_{ij})} \\
 &+ \left(\frac{\alpha}{4\pi} \right)^2 \frac{1}{2\epsilon} \left[\left(\frac{-s}{\mu^2} \right)^\epsilon \Delta K(\epsilon, 0, s) - \Delta K(2\epsilon, 0, s) \right] e^2 Q_i Q_j b_{\text{QED}}^{(1)}
 \end{aligned}$$

Two-loop NLL result for $f_1 f_2 \rightarrow f_3 \dots f_n$

$$\begin{aligned}
& \text{tree} + \sum_{i,j} \frac{1}{2} \text{diagram}_1 + \sum_{i,j,k,l} \frac{1}{2} \left[\text{diagram}_2 + \text{diagram}_3 \right] + \text{diagram}_4 \\
& + \text{diagram}_5 + \text{diagram}_6 + \frac{1}{2} \text{diagram}_7 + \text{diagram}_8 + \frac{1}{6} \text{diagram}_9 + \frac{1}{8} \text{diagram}_{10} =
\end{aligned}$$

$$= \exp \left[\sum_{j < i} \text{diagram with wavy line } \Delta\gamma \right] \exp \left[\sum_{j < i} \text{diagram with wavy line } W, Z, \gamma \right] \underbrace{\left[1 + \sum_{j < i} \text{diagram with wavy line } \Delta Z \right]}_{\text{tree}} \text{diagram of a tree with nodes } 1, 2, 3, \dots, n$$

Mixing correction depending on Z-W mass difference

$$\text{Diagram: a vertex splitting into two lines labeled } i \text{ and } j, \text{ with a wavy line labeled } \Delta Z \text{ connecting them.} = - \left(\frac{\alpha}{4\pi} \right) I_i^Z I_j^Z \ln \left(\frac{M_Z^2}{M_W^2} \right) [2L + 2L^2 \varepsilon + L^3 \varepsilon^2] \Rightarrow \mathcal{O}(10^{-3}) \text{ effect at two loops}$$

$\Rightarrow \mathcal{O}(10^{-3})$ effect at two loops

Two-loop NLL result for $f_1 f_2 \rightarrow f_3 \dots f_n$

$$\begin{aligned}
 & \text{tree} + \sum_{i,j} \frac{1}{2} \text{diagram} + \sum_{i,j,k,l} \frac{1}{2} \left[\text{diagram} + \text{diagram} \right] + \text{diagram} \\
 & + \text{diagram} + \text{diagram} + \frac{1}{2} \text{diagram} + \text{diagram} + \frac{1}{6} \text{diagram} + \frac{1}{8} \text{diagram} = \\
 & = \exp \left[\sum_{j < i} \text{diagram} \right] \exp \left[\sum_{j < i} \text{diagram} \right] \left[1 + \sum_{j < i} \text{diagram} \right] \text{tree}
 \end{aligned}$$

- massless-fermion scattering well understood
- method developped for general processes with massive fermions and bosons
- work in progress ...

Conclusions

Hard reactions at $Q^2 \sim 1 \text{ TeV}^2$ receive large $\ln(s/M_W^2)$ EW corrections

- one loop $\gg 10\%$
- two loops $\gg 1\%$ (important for LHC/ILC precision tests and NP searches)

Systematic asymptotic expansions

- strong simplification, process-independent treatment
- good precision (requires subleading logarithms!)

Status and future challenges

- NLLs for any process within reach
- Next important challenges: NNLLs, hard photon bremsstrahlung ...

pp $\rightarrow$ WW, WZ, ZZ $\rightarrow$ 4 leptons

	pp $\rightarrow$ WZ $\rightarrow l\nu_l l' \bar{l}'$		pp $\rightarrow$ ZZ $\rightarrow l\bar{l} l' \bar{l}'$		pp $\rightarrow$ WW $\rightarrow l\bar{\nu}_l \nu_{l'} \bar{l}'$	
$M_{\text{inv}}^{\text{cut}}(\text{leptons})[\text{GeV}]$	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]
500	-7.4	5.4	-15.0	8.5	-13.8	2.6
600	-9.5	7.5	-18.3	11.9	-15.9	3.7
700	-10.9	9.9	-21.0	15.7	-18.1	4.9
800	-13.3	12.8	-23.8	20.1	-20.2	6.5
900	-15.1	16.2	-26.1	25.3	-22.0	8.3
1000	-16.7	20.2	-28.1	31.2	-23.4	10.4

estimate based on $L = 100\text{fb}^{-1}$ and final states with $l, l' = e$ or μ

Accomando, Denner, Kaiser (2005)

Electroweak corrections (Δ_{EW}) vs statistical error ($\Delta_{\text{stat}} = 1/\sqrt{2L\sigma_0}$)

- Δ_{EW} and Δ_{stat} grow with $M_{\text{inv}}(\text{leptons})$
- $|\Delta_{\text{EW}}| \gtrsim \Delta_{\text{stat}}$ up to $M_{\text{inv}} \sim 1 \text{ TeV}$

Double logarithms beyond one loop

QED vertex corrections for $s \gg \lambda^2$

$$\text{Diagram: wavy line to vertex, vertex to two fermion lines, internal wavy line labeled } \lambda \text{ between vertices} = -\frac{\alpha}{4\pi} \ln^2 \left(\frac{s}{\lambda^2} \right)$$

Double logarithms beyond one loop: exponentiation

$$\dots + \frac{1}{2} \left(\text{Diagram} \right)^2 + \frac{1}{3!} \left(\text{Diagram} \right)^3 + \dots = \exp \left(\text{Diagram} \right)$$

Sudakov (1956)

Back-of-the-envelope estimate of two-loop electroweak logs at $\sqrt{s} \simeq 1 \text{ TeV}$

$$\left(\frac{\delta\sigma}{\sigma_{\text{Born}}} \right)_{2 \rightarrow 2} \simeq \frac{1}{2} \left(\sum_{j \neq i} \text{Diagram: vertex to two fermion lines } i, j, \text{ internal wavy line labeled } W, Z, \gamma \right)^2 \simeq \frac{\alpha^2}{2\pi^2 s_W^4} \ln^4 \frac{s}{M_W^2} \simeq 3.5\%$$

well beyond 1%

Cancellation mechanism for non-factorizable contributions

Soft-collinear fermion-boson vertices (Dirac structure disappears)

$$\lim_{q_k^\mu \rightarrow x_k p_i^\mu} \text{Diagram} = G_{\mu_1 \dots \mu_n}^{\bar{V}_1 \dots \bar{V}_n}(-q_1, \dots, -q_n, p_i + \tilde{q}_n) u(p_i, \kappa_i) \times \frac{-2eI_i^{V_n}(p_i + \tilde{q}_n)^{\mu_n}}{(p_i + \tilde{q}_n)^2} \dots \frac{-2eI_i^{V_1}(p_i + q_1)^{\mu_1}}{(p_i + q_1)^2}$$

Collinear Ward identities for spontaneously broken non-abelian theories

$$\lim_{q^\mu \rightarrow x p_i^\mu} q^\mu \times \left[\text{Diagram 1} - \text{Diagram 2} - \sum_{\substack{j=1 \\ j \neq i}}^n \text{Diagram 3} \right] = 0$$

Denner, S.P. (2001)

derived from BRS symmetry and valid for arbitrary processes

Origin of $1/\varepsilon$ and $\ln(Q^2/M^2)$ singularities

$$G \propto \int_0^1 d^I \vec{\alpha} \delta(1 - \sum_{r=1}^I \alpha_r) \frac{\Gamma(e)}{[\mathcal{U}(\vec{\alpha})]^e [\mathcal{F}(\vec{\alpha})]^f}$$

Polynomials ($\mathcal{T}$ = trees, $\mathcal{C}$ = cuts)

$$\mathcal{U}(\vec{\alpha}) = \sum_{\mathcal{T}} \alpha_{\mathcal{T}_1} \dots \alpha_{\mathcal{T}_L}$$

$$-\mathcal{F}(\vec{\alpha}) = \sum_{\mathcal{C}} s_{\mathcal{C}} \alpha_{\mathcal{C}_1} \dots \alpha_{\mathcal{C}_{L+1}} - \mathcal{U}(\vec{\alpha}) \sum_{r=1}^I \alpha_r M_r^2 + i\varepsilon$$

UV and mass singularities ($s_{\mathcal{C}} = s, t, u < 0$)

$$\mathcal{U}(\vec{\alpha}) = 0 \Rightarrow \text{UV sing.} \qquad \mathcal{F}(\vec{\alpha}) = 0 \Rightarrow \text{mass sing.}$$

Singular regions ($\mathcal{U} = 0, \mathcal{F} = 0$)

$$\{\vec{\alpha} | \alpha_{i_1} = \dots = \alpha_{i_n} = 0\}$$

Crucial for factorization of singularities in FP space!

Step 2: **Sector decomposition**

Goal: **factorization** of mass singularities from $\mathcal{F}(\vec{\alpha})$

$$\int_0^1 \frac{d^{I-1} \vec{\alpha}}{\underbrace{[Q^2 \mathcal{P}(\vec{\alpha}) + M^2 \mathcal{R}(\vec{\alpha})]^\mathcal{f}}_{\mathcal{F}(\vec{\alpha})} \dots} \Rightarrow \int_0^1 \frac{d^{I-1} \vec{\alpha}}{[Q^2 \underbrace{\hat{\mathcal{P}}(\vec{\alpha})}_{\neq 0} \alpha_1 \dots \alpha_k + M^2 \hat{\mathcal{R}}(\vec{\alpha})]^\mathcal{f} \dots}$$

Sector decomposition for **overlapping singularities** $\mathcal{P}(\vec{\alpha})|_{\alpha_1=\dots=\alpha_k=0} = 0$

(A) **partition** of $[0, 1]^{I-1}$ into sectors $\Omega_1, \dots, \Omega_k$

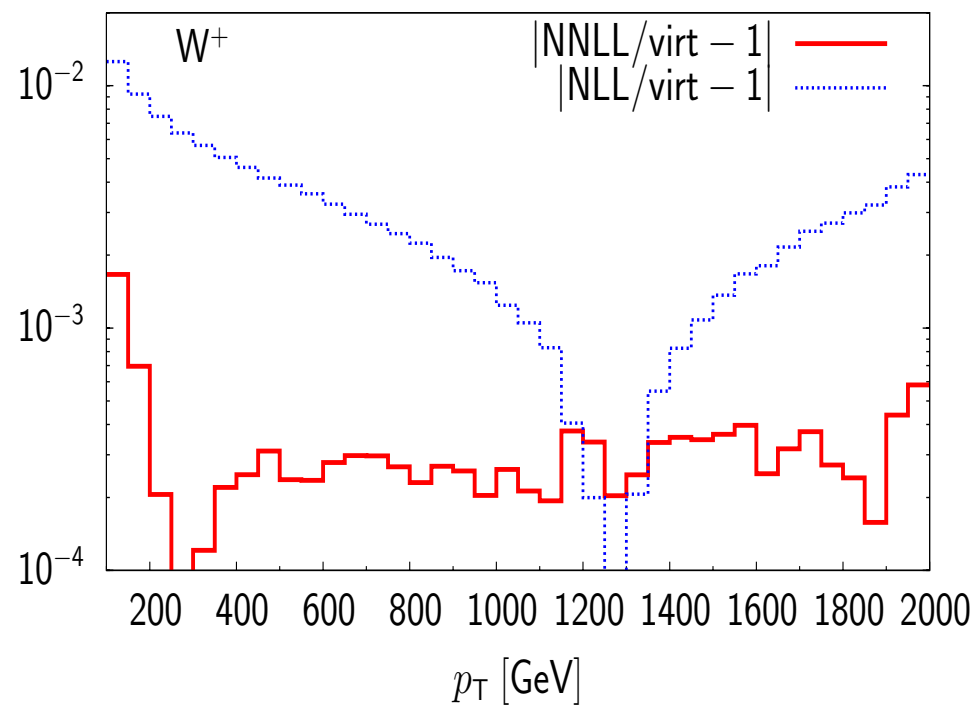
$$\Omega_j = \{\vec{\alpha} | \alpha_1, \dots, \alpha_k \leq \alpha_j\}$$

(B) **remapping** $\Omega_j \rightarrow [0, 1]^{I-1}$ yields **factorization** in Ω_j -sector

$$\alpha_k \rightarrow \alpha_k \alpha_j \text{ for } k \neq j \quad \Rightarrow \quad \mathcal{P}(\vec{\alpha}) \rightarrow \alpha_j \hat{\mathcal{P}}_j(\vec{\alpha})$$

(C) **iterate** until $\hat{\mathcal{P}}_j(\vec{\alpha}) \neq 0$

Precision of NLL and NNLL approximations for $pp \rightarrow W+\text{jet}$



$$\text{NLL-error} \lesssim 10^{-2}, \text{NNLL-error} \lesssim 10^{-3}$$

Dominant two-loop contributions for $\bar{q}q \rightarrow Zg$

Include **LL** and **NLL** terms

$$\mathbf{X}_1 = \ln^4 \left(\frac{\hat{s}}{M_W^2} \right) - 6 \ln^3 \left(\frac{\hat{s}}{M_W^2} \right)$$

$$\mathbf{X}_2 = \ln^4 \left(\frac{\hat{t}}{M_W^2} \right) + \ln^4 \left(\frac{\hat{u}}{M_W^2} \right) - \ln^4 \left(\frac{\hat{s}}{M_W^2} \right)$$

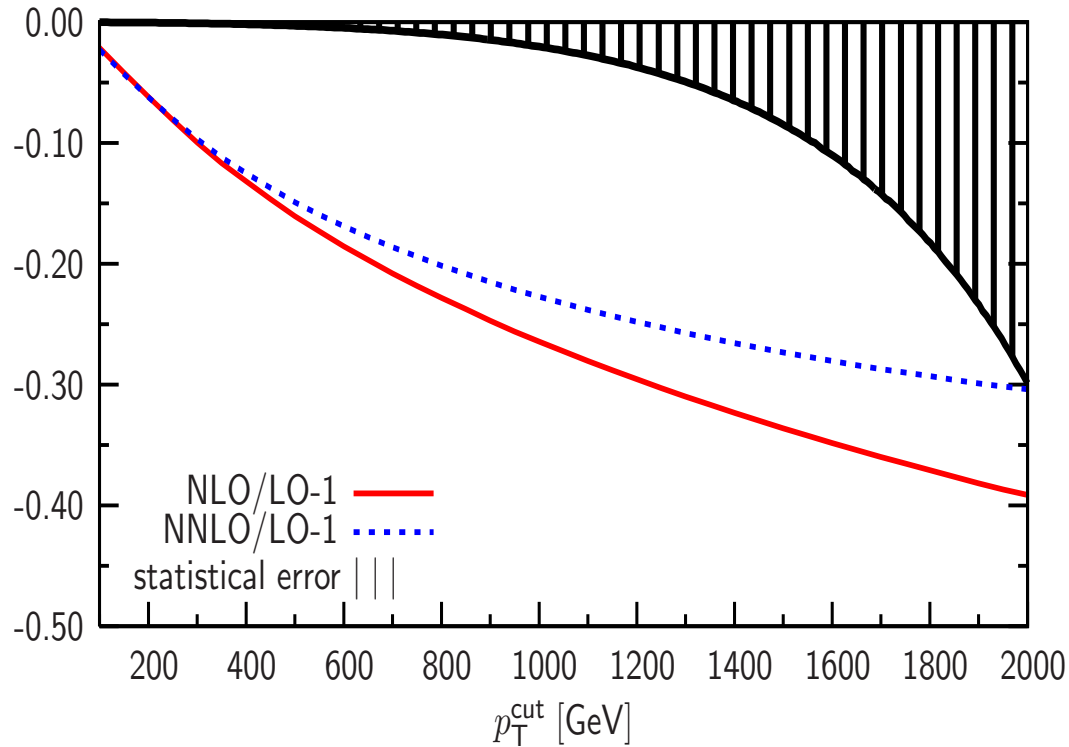
$$\mathbf{X}_3 = \ln^3 \left(\frac{\hat{s}}{M_W^2} \right)$$

derived from general two-loop results for EW logarithms (see later)

$$\begin{aligned} \overline{\sum} |\mathcal{M}_2|^2 = & \overline{\sum} |\mathcal{M}_1|^2 + 2\alpha^3 \alpha_S (N_c^2 - 1) \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} \sum_{\lambda=L,R} \left\{ \frac{1}{2} \left(I_{q\lambda}^Z C_{q\lambda}^{\text{ew}} + \frac{c_W}{s_W^3} T_{q\lambda}^3 \right) \right. \\ & \times \left[I_{q\lambda}^Z C_{q\lambda}^{\text{ew}} \mathbf{X}_1 + \frac{c_W}{s_W^3} T_{q\lambda}^3 \mathbf{X}_2 \right] - \frac{T_{q\lambda}^3 Y_{q\lambda}}{8s_W^4} \mathbf{X}_2 + \frac{1}{6} I_{q\lambda}^V \left[I_{q\lambda}^Z \left(\frac{b_1}{c_W^2} \left(\frac{Y_{q\lambda}}{2} \right)^2 + \frac{b_2}{s_W^2} C_{q\lambda} \right) + \frac{c_W}{s_W^3} T_{q\lambda}^3 b_2 \right] \mathbf{X}_3 \left. \right\} \end{aligned}$$

Melles (2001); Denner, Melles, S.P. (2003)
Kühn, Kulesza, S.P., Schulze (2005)

1- and 2-loop corrections to $\sigma(p_T > p_T^{\text{cut}})$ for $pp \rightarrow Z + \text{jet}$



$\overline{\text{MS}}$ input: $\alpha_S(M_Z^2) = 0.13$, $\alpha = 1/128.1$, $s_w^2 = 0.2314$

PDFs: LO MRST2001, $\mu_F = \mu_R = p_T$

Estimated statistical error

- $\mathcal{L} = 300 \text{ fb}^{-1}$, $Z \rightarrow \text{leptons}$
- $(\Delta\sigma/\sigma)_{\text{stat}} \sim 2\%$ at 1 TeV

Size of corrections at $p_T \sim 1 \text{ TeV}$

- 1-loop: $-26\% \simeq -13\sigma_{\text{stat}}$
- 2-loop: $+4\% \simeq +2\sigma_{\text{stat}}$

Resummation formula based on IREE approach

Double exponentiation resulting from $M_\gamma \ll M_W \sim M_Z$

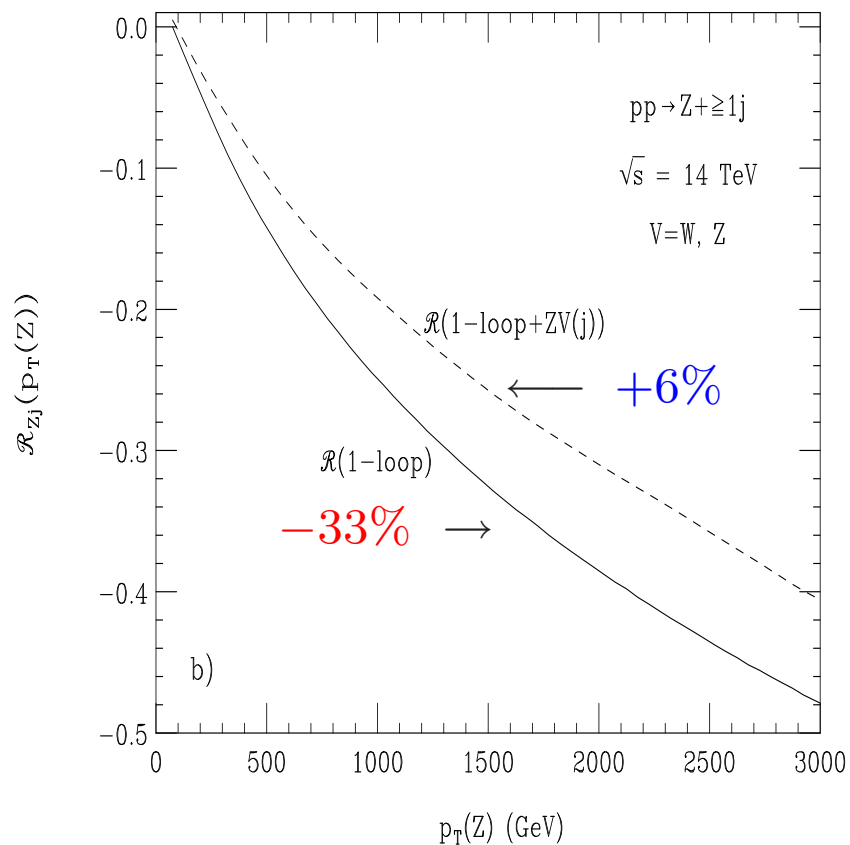
$$\begin{array}{c}
 \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \vdots \\ \diagdown \\ n \end{array} = \exp \left[\underbrace{\sum_{j < i} \begin{array}{c} i \\ \diagup \\ \text{---} \Delta\gamma \text{---} \\ \diagdown \\ j \end{array}}_{\substack{\ln(M_W/M_\gamma) \equiv \\ \text{QED IR sing.}}} \right] \exp \left[\underbrace{\sum_{j < i} \begin{array}{c} i \\ \diagup \\ \text{---} W, Z, \gamma \text{---} \\ \diagdown \\ j \end{array}}_{\substack{\ln(s/M_W^2) \equiv \text{SU}(2) \times \text{U}(1) \\ \text{theory with } M_\gamma = M_W}} \right] \begin{array}{c} 1 \\ \diagdown \\ \text{tree} \\ \diagup \\ 2 \end{array} \begin{array}{c} 3 \\ \diagup \\ \vdots \\ \diagdown \\ n \end{array}
 \end{array}$$

Existing results

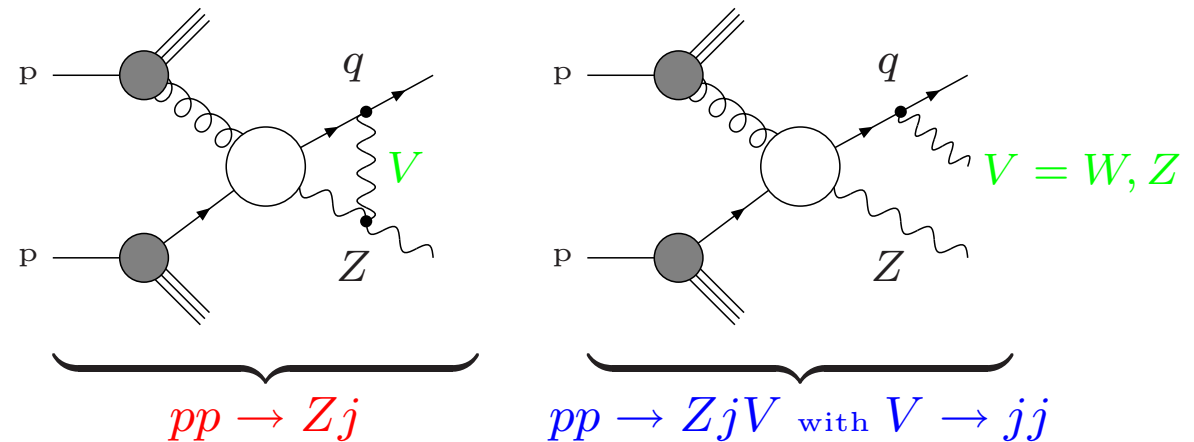
$$\alpha^2 \left[\underbrace{\underbrace{C_4 \ln^4 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Fadin, Lipatov, Martin,} \\ \text{Melles (2000)}}} + \underbrace{C_3 \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{Melles (2001–2004)}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_2 \ln^2 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Kühn, Moch, Penin,} \\ \text{Smirnov (2000–2003)}}} + \underbrace{C_1 \ln^1 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Jantzen, Kühn, Penin,} \\ \text{Smirnov (2004, 2005)}}}}_{\text{massless } f\bar{f} \rightarrow f'\bar{f}' \text{ processes}} \right]$$

Real W and Z emission for $pp \rightarrow Zj$ [Baur (2006)]

“ Since the number of jets is not fixed in a measurement of the Z boson p_T distribution, $\mathcal{O}(\alpha_s\alpha^2)$ ZVj production with $V \rightarrow jj$ has to be included when calculating weak radiative corrections ”



Virtual and real $\mathcal{O}(\alpha)$ corr. to $pp \rightarrow Zj$



- W, Z emission can be non-negligible and partially cancel EW virtual corrections
- depends on observable definition and can be reduced by jet veto