CP property of the Higgs at the ILC.

- ♢ Introduction.
- ♢ Two issues:
  - i Determination of CP for a Higgs which is a CP eigen-state.
  - ii Determination of CP mixing in the Higgs sector for a Higgs with indeterminate CP.

R.G, S.Kraml, M.Krawczyk, D.J.Miller, P.Niezurawski and A.F.Zarnecki in G. Weiglein et al., Phys. Rept. 426 (2006) 47 and hep-ph/0404024.

hep-ph/0608079 CPNSH report.

R.M. Godbole, Pramana **67** (2006) 835.

A. Djouadi, hep-ph/0503172, 0503173.

Bhupal Dev,A.D.,R.G.,Muhelleitner, Rindani hep-ph/0707.2848

R.G., D. Miller and M. Muehlleitner: hep-ph/0708.0458

Importance of Studies of CP Properties of Higgs Boson

- Just the discovery of the Higgs boson is not sufficient to validate the minimal SM.
- In SM, the only fundamental neutral scalar is a $J^{PC} = 0^{++}$ state arising from an $SU(2)_L$ doublet with $Y = +1$.
- Various extensions of the SM can have several Higgs bosons with different CP properties : e.g. MSSM has two CP-even and one CP-odd states.
- Therefore, should a neutral spin-0 particle be detected, a study of its CP-properties would be essential to establish it as *the* SM Higgs boson.
- To study the New Physics effects beyond SM, we need to establish the CP eigenvalues for the Higgs states if CP is conserved, and measure the mixing between CP-even and CP-odd states if it is not.
- CP violation in the Higgs sector can be an alternative source of CP violation beyond the SM, required to explain the observed baryon asymmetry in our universe. [Accomando et al., *CERN 2006-009* (2006)]

Effect of SUSY $\not{CP}$ on Higgs phenomenology

MSSM $\not{CP}$ phases $\Rightarrow$ $\not{CP}$ in the Higgs sector:

CP conserving MSSM Three Neutral Higgses h,H CP-even A CP-odd

CP violation : ϕ_1, ϕ_2, ϕ_3 no fixed CP property

$m_{\phi_1} < m_{\phi_2} < m_{\phi_3}$

Sum rules exist for $\phi_i f\bar{f}$, $\phi_i VV$

(A. Mendez and A. Pomarol, J.Gunion, H. Haber and J. Wudka, B.Grzadkowski, J.Gunion and J. Kalinowski.)

$$g^2_{\phi_i WW} + g^2_{\phi_j WW} + g^2_{\phi_k WW} = g^2 m^2_W, i \neq j \neq k$$

First proposed in a model independent way.

The h, H, A now all mix and share the couplings with vector boson pair VV. Will affect production rates.

Predictions in terms of SUSY $\not{CP}$ phases in the MSSM for this mixing.

CP Study in the Higgs sector

(R.G., Kraml, Krawczyk,Miller et al in LHC/LC study group report.)

1. Determination of the CP properties of the Spin 0 particle(s) which we hope will be discovered at the future colliders.

2. Determination of the CP mixing if discovered scalars ($\simeq$ Higgses) **NOT** CP eignestates.

Establish tensor structure for $\phi_i f \bar{f}$, $\phi_i VV$ vertex.

ϕ_i : a generic Higgs.

General Strategy for CP determination:

Couplings with pair of gauge bosons ($ZZ/\gamma\gamma/WW$) and the pair of heavy fermions (t/τ) are the ones which are useful.

Study $\not{C}\not{P}$ in a model indpendent way (most studies so far)

$$\begin{aligned}
\phi_i f\bar{f} &: \quad -\bar{f}(s_f + i p_f \gamma_5)\frac{g m_f}{2 m_W}, \\
VV\phi_i &: \quad a_V \frac{g m_V^2}{m_W} g_{\mu\nu}(\mathrm{V = W/Z, g : tree/loop\ level}) \\
&: \quad \eta \epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma / m_Z^2 (\mathrm{loop\ level})
\end{aligned}$$

1. SM: $s_f = c_V = 1, p_f = 0$,i = 1.
2. $s_f = c_V = 0$ and $p_f \neq 0$ for the CP odd Higgs, for general CP conserving multi-Higgs models.
3. Pseudoscalar $\epsilon^{\mu\nu\rho\sigma}$: only at loop level in MSSM and CP conserving 2HDM.
4. Generically CP mixing is a loop effect, hence small.

Collider	CP determination	Measurement of Mixing
ILC	$f\bar{f}$Higgs final state $VV, f\bar{f}$ final states	$f\bar{f}$ Higgs final state $VV, f\bar{f}$ final state
$\gamma\gamma$	VV final state VV fusion	Best for study of mixing

VV and $f\bar{f}$ final state angular distributions show striking differences due to the differences in the tensor structure.

Most important advanatge for $t\bar{t}H$ final state and $\gamma\gamma$ colliders: Production channel treats both the scalar and the pseudscalar the same way. Then use all the same methods as at other colliders. The most unambigious way to measure CP mixing.

$\gamma\gamma$ colliders possible with backscattered lasers at a parent e^+e^- collider. Likely to be in the far future. M. Krawczyk's talk?

- Use kinematic distribution of the decay products of the Higgs: $H \to f\bar{f}(f = t, \tau)$, $H \to ZZ(Z^*) \to f\bar{f}f'\bar{f}'$.

- What distributions: Angular distributions, invariant mass distributions, angular correlations.

- Kinematics of the production process, threshold rise.

- Spin information of the fermions produced in the decay of Higgs or the fermions which are produced in association with the Higgs.

- For the τ decay products carry the spin information of the decaying τ. Due to its large decay width ($\Gamma_t \sim 1.5$ GeV), top also decays much before hadronization; hence its spin information is translated to the decay distribution before being contaminated by hadronisation effects. Hence $\phi \to t\bar{t}, \phi \to \tau^+\tau^-$ and $e^+e^- \to t\bar{t}\phi$ carry information on CP character of Φ.

- The decay lepton angular distribution for the t is independent of any non-standard effects in the top decay vertex. Thus this distribution is a pure probe of new physics associated with the t-production [e.g. Godbole, Rindani, and Singh, *JHEP* **12**, 021 (2006)]. Lepton angular distribution a good polariometer. Measuring decay lepton angular distribution asymmetries can give information on produced top polarisation asymmetries.

For a light Higgs, most promising, at an ILC, is to exploit the $ZZ(Z^*)$ coupling for production , $H \to ZZ^{(*)} \to f\bar{f}f'\bar{f}'$, $H \to \tau^+\tau^-$.

a)Energy dependecne of the total production cross-section in Higgsstahlung.

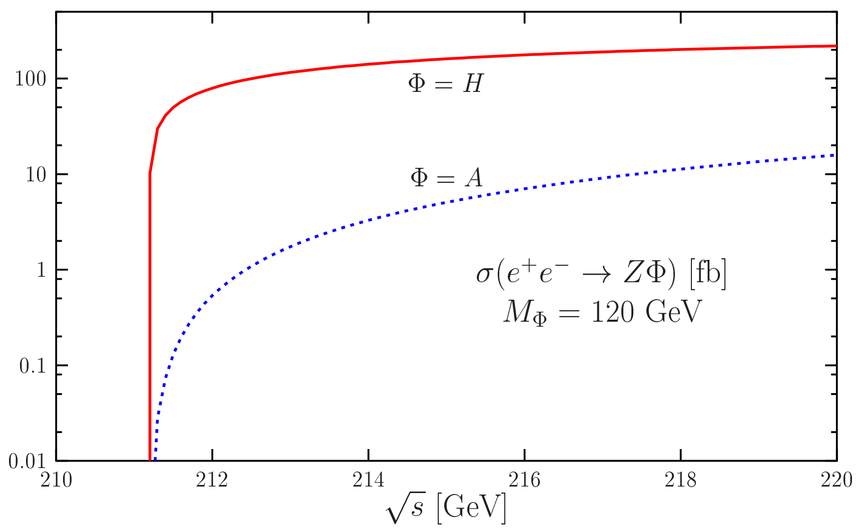
b)Production angular distribution.

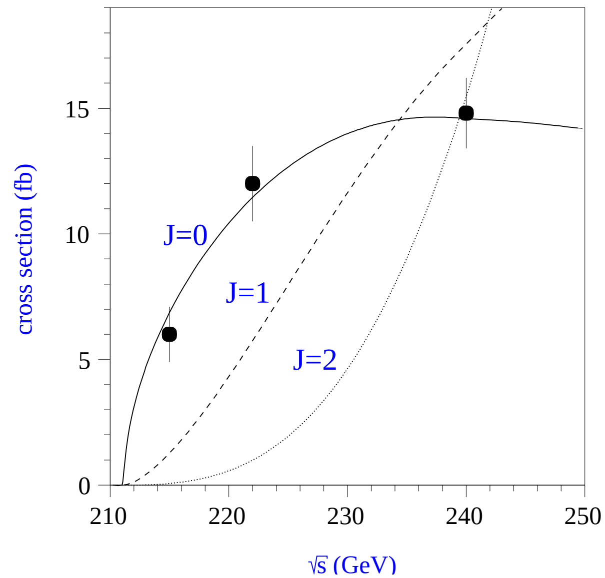
c)Angular correlations.

Zerwas, Djouadi, Barger, Kniehl, Keung, Choi, Miller, Osland,Kraemer,Was, Desch, Worek,Choi, J.S. Lee, Pilaftsis....

$$\sigma(e^+e^- \to HZ) \sim \lambda^{1/2} \sim \sqrt{s - (M_H + M_Z)^2}$$

$$\sigma(e^+e^- \to ZA) = \eta^2 \frac{G_\mu^2 M_Z^6}{48\pi M_A^4} (\hat{a}_e^2 + \hat{v}_e^2) \frac{\lambda^{3/2}}{(1 - M_Z^2/s)^2}$$

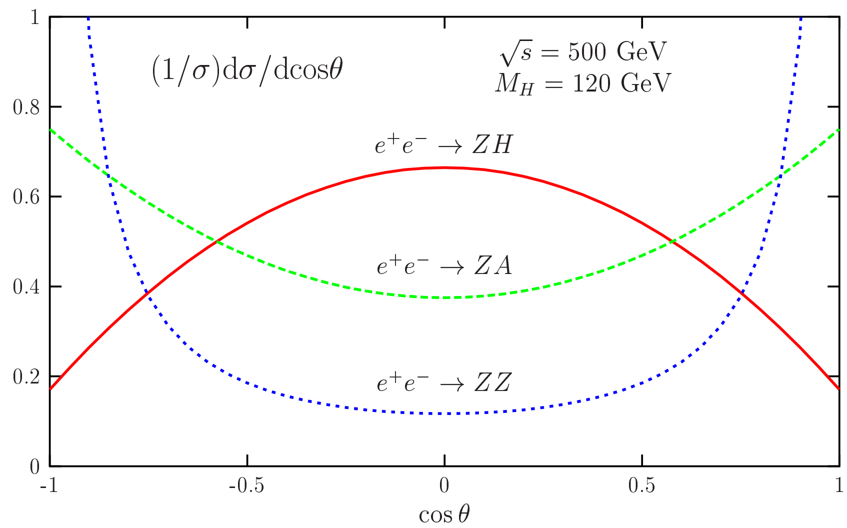


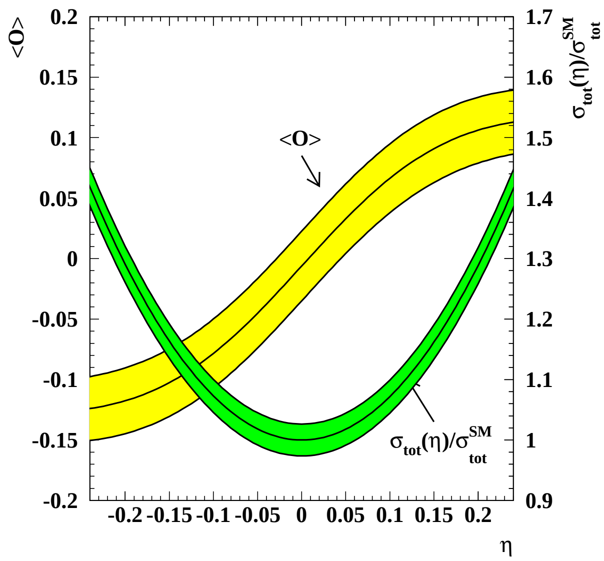


Threshold rise can determine spin, and can discriminate against $0^-, 1^-$ etc. Angular distributions sensitive to Parity as well.

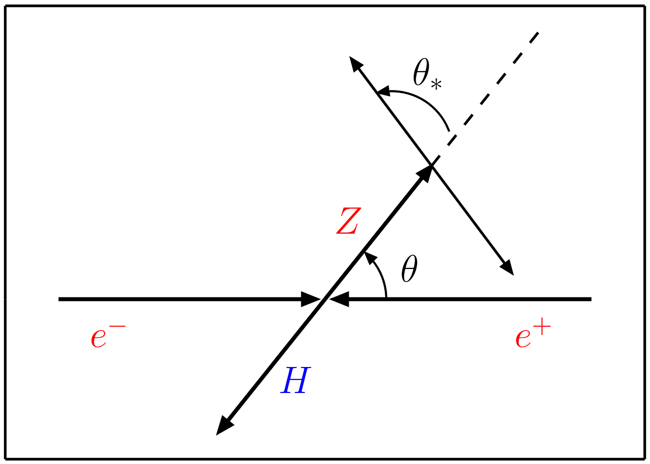
$$\frac{\mathrm{d}\sigma(e^+e^-\to ZH)}{\mathrm{d}\cos\theta} \sim \lambda^2 \sin^2\theta + 8M_Z^2/s$$

$$\frac{\mathrm{d}\sigma(e^+e^-\to ZA)}{\mathrm{d}\cos\theta} \sim 1 + \cos^2\theta$$





Even CP mixing can be measured using this. Next is angular correlations and azimuthal distributions.

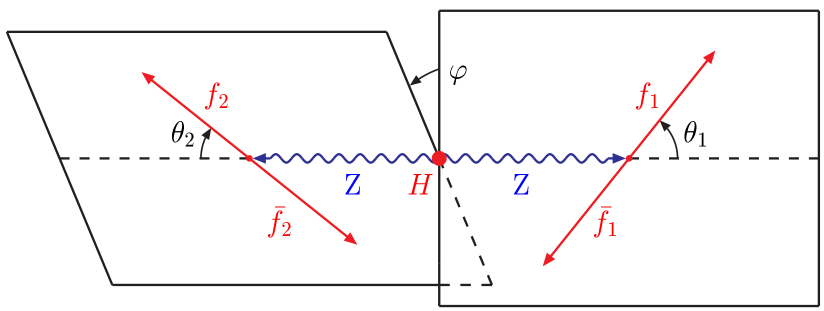


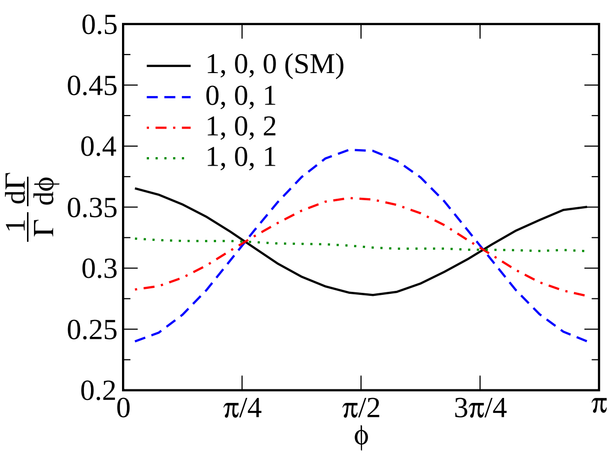
$$\frac{\mathrm{d}\sigma(e^+e^-\to ZH)}{\mathrm{d}\phi_*} \sim 1 + a_1 \cos\phi_* + a_2 \cos 2\phi_*$$

$$\frac{\mathrm{d}\sigma(e^+e^-\to ZA)}{\mathrm{d}\phi_*} \sim 1 - \frac{1}{4}\cos 2\phi_*$$

ϕ_* azimuthal angle of the plane of $Z \to f\bar{f}$ decay and Higgs decay products.

The definition of the polar angles θ_i $(i = 1, 2)$ and the azimuthal angle φ for the sequential decay $H \to Z^{(*)}Z \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ in the rest frame of the Higgs boson.





Need to distinguish between f_1 and $\bar{f}_1$. One Z decays to $f_1\bar{f}_1$ and other two $f_2\bar{f}_2$.

In the SM

$$\frac{d\Gamma}{d\varphi} \sim 1 + A\cos\varphi + B\cos 2\varphi$$

A, B are funtions of M_H, M_Z. the ϕ dependence will vanish for larger Higgs masses.

For CP odd case

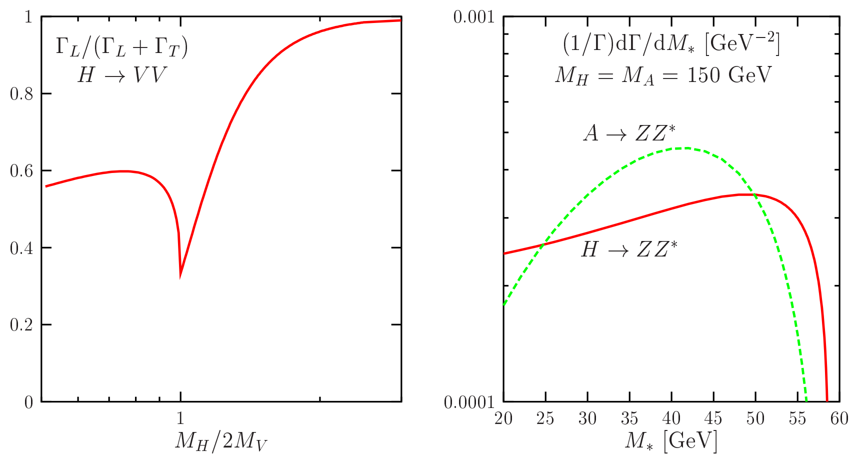
$$\frac{d\Gamma}{d\varphi} \sim 1 - \frac{1}{4}\cos 2\varphi$$

A decays only into transverse V and H into both transverse and longtudinal fraction changing with V^* for a fixed M_ϕ.

$$\frac{d\Gamma(H\to VV^*)}{dM_*^2} = \frac{3G_\mu^2 M_V^4}{16\pi^3 M_H}\delta'_V \; \frac{\beta_V (M_H^4 \beta_V^2 + 12 M_V^2 M_*^2)}{(M_*^2 - M_V^2)^2 + M_V^2 \Gamma_V^2}$$

with $\beta_V^2 = [1-(M_V+M_*)^2/M_H^2][1-(M_V-M_*)^2/M_H^2]$.

$$\frac{d\Gamma(A\to VV^*)}{dM_*^2} = \frac{3G_\mu^2 M_V^6}{8\pi^3 M_A}\delta'_V \eta^2 \frac{M_*^2 \beta_V^3}{(M_*^2 - M_V^2)^2 + M_V^2 \Gamma_V^2}$$



$\Gamma_{\text{Born}}(H \to f\bar{f}) = \frac{G_\mu N_c}{4\sqrt{2}\pi} M_H\, m_f^2\, \beta_{\mathbf{f}}^{\mathbf{3}}$

$\Gamma_{\text{Born}}(A \to f\bar{f}) = \frac{G_\mu N_c}{4\sqrt{2}\pi} M_H\, m_f^2\, \beta_{\mathbf{f}}$

If spins of $f\bar{f}$ are $s, \bar{s}$ respectively,

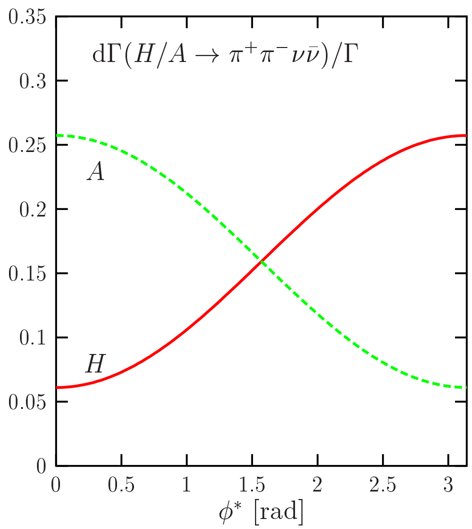
Use $e^+e^- \to Z^* \to ZH \to Z\tau^+\tau^-$

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}(s,\bar{s}) \;=\; & \frac{\beta_f}{64\pi^2 M_\Phi}\big[(|a|^2+|b|^2)\big(\frac{1}{2}M_\Phi^2 - m_f^2 + m_f^2 s\cdot\bar{s}\big) \\ & +(|a|^2-|b|^2)\big(p_+\cdot s\, p_+\cdot\bar{s} - \frac{1}{2}M_\Phi^2 s\cdot\bar{s} + m_f^2 s\cdot\bar{s} - m_f^2\big) \\ & -\mathrm{Re}(ab^*)\epsilon_{\mu\nu\rho\sigma}p_+^\mu p_-^\nu s^\rho \bar{s}^\sigma - 2\mathrm{Im}(ab^*) m_f p_+\cdot(s+\bar{s})\big]\end{aligned}$$

$\Gamma(H/A \to t\bar{t}) \propto 1 - s_z\bar{s}_z \pm s_\perp \bar{s}_\perp$

Possible to probe CP using the tau decay products.

(Was, Desch and Worek: analysis for mixing determination as well making fits to the distribution.)



ϕ angle between the decay planes of the $\tau^{\pm}$. Asymmetry is clearly visible.

The parity can be also determined by looking at the the distribution in the angle between the pions into which the $\tau^{\pm}$ decay. This in turn determined by the polarisation of the $\tau^{\pm}$ and that in turn by the CP.

Worek et al showed one could be sensitive to an angle of six degree.

$H \to t\bar{t}$ will offer information only for heavy Higgs.

All the above processes use ϕZZ coupling for prooduction. Which means for a pseudoscalar the strength is necessarily small as loops are involved. For a state of mixed CP, only the CP-even part gets projected out in production. This is true of all the various studies suggested above.

$t\bar{t}\phi$ production couples democratically.

Gunion and collaborators studied optimal observable technique to study CP property of the Higgs and concluded that with a high luminosity it should be possible to measure even a mixing of a few degrees. Slice the phase space region and use the kinematical distributions of the particles expected for the signal in an optimal way.However, the physics is somewhat obscured by the optimal observable technique used.

- We Point out a simple way to discriminate CP even and CP odd case.

- Specifically use $t\bar{t}\phi$ coupling is

$$g_{t\bar{t}\phi} = -ig_2 \frac{m_t}{2m_W}(a + ib\gamma_5) \ ,$$

  where $a = S_f, \ b = P_f$.

- We take the $ZZ\phi$ Coupling to be similar to the SM case:

$$(g_{ZZ\phi})_{\mu\nu} = -ic \frac{g_2 m_Z}{\cos\theta_W} g_{\mu\nu}$$

  we will see that the effect of this term will be negligible here. Can be probed using $e^+e^- \to Z\phi$ (eg. Phys. Rev. D 06, Biswal et al)

- In the SM, $a = 1 = c$ and $b = 0$. A model-independent way of parametrization can be $|a|^2 + |b|^2 = 1$. We have taken $c = a$.

- Moreover, we treat $a, \ b, \ c$ to be all real.

- Hence only one CP-violating term ab and only independent parameter b.

- In principle, in a specifc model we may have predictions for a, b, c: e.g. THDM and CP-violating MSSM.

- Recall the generalized $t\bar{t}\phi$ and $ZZ\phi$ couplings:

$$g_{t\bar{t}\phi} = -ig_2 \frac{m_t}{2m_W}(a + ib\gamma_5) \ ,$$

with our choice of parametrization $|a|^2 + |b|^2 = 1$; $a,\ b$ both being real.

- Hence only independent parameter b with $a = \sqrt{1-b^2}$.

- We have studied the sensitivity of b to simple observables such as cross section and polarization asymmetry, with and without polarized beams.

- The Polarization Asymmetry for top-quark is given by

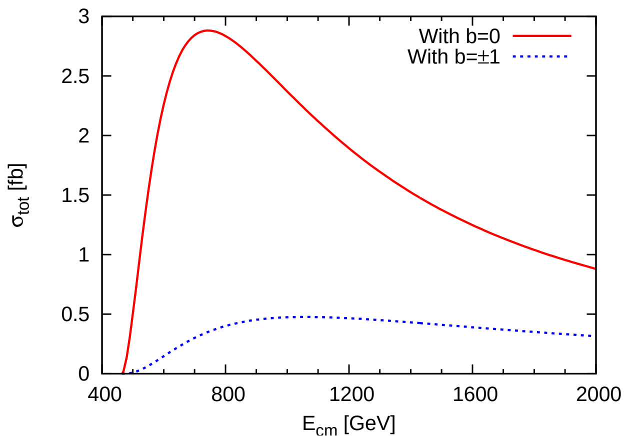
$$P_t = \frac{\sigma(t_L) - \sigma(t_R)}{\sigma(t_L) + \sigma(t_R)} \qquad \text{(with unpolarized initial beams),}$$

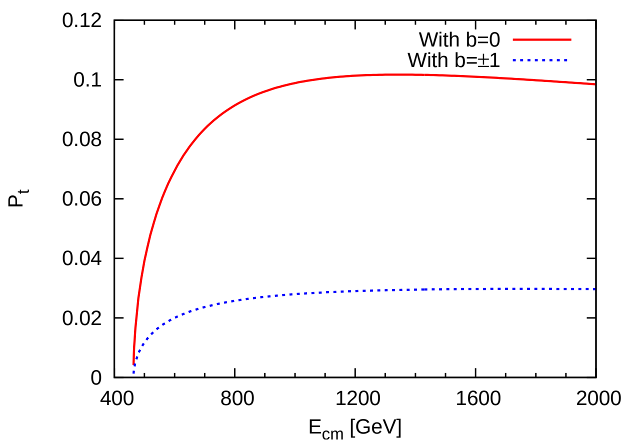
$$P_t^e = \frac{\sigma_t^e(t_L) - \sigma_t^e(t_R)}{\sigma_t^e(t_L) + \sigma_t^e(t_R)} \qquad \text{(with polarized initial beams),}$$

$$\text{with} \quad \sigma_{\text{tot}}(\text{unpolarized}) = \frac{1}{4}\,[\sigma_{RL} + \sigma_{LR}],$$

$$\text{and} \quad \sigma_t^e(\text{polarized}) = \frac{1+P_{e^-}}{2}\,\frac{1-P_{e^+}}{2}\,\sigma_{RL} + \frac{1-P_{e^-}}{2}\,\frac{1+P_{e^+}}{2}\,\sigma_{LR}$$

($\sigma_{RL(LR)}$ corresponds to the completely polarized $e^-_{R(L)}e^+_{L(R)}$ beams)





Threshold dependence very different for scalar and pseudoscalar. Steep dependence (S vs P wave). Define $\rho = 1 - 2m_t/\sqrt{s} - M_\Phi/\sqrt{s}$

$$F_1^H = -F_2^H \simeq 12\left[m_t^2/(M_H\sqrt{s})\right]^{3/2} \ \rho^2$$

$$F_1^A = -F_2^A \simeq 4\left[m_t^4/(M_A s\sqrt{s})\right]^{1/2} \ \rho^3.$$

May be just two measurements, at 500 and (say) 800, would see the difference. For $M_\phi = 120$ GeV, the ratios for H and A are 7.5 and 63, as $\sqrt{s}$ changes from 500 to 800 GeV.

Recall: radiative corrections are also substantia. So taking ratios is a good idea.

Polarisation shows similar energy dependence and is again different for H(b=0) and A(b=1). Wang et al calculated polrisation, did not really use it.

- Δb is the sensitivity at $b = b_0$ if for an observable $O(b)$,

$$|O(b) - O(b_0)| = \Delta O(b_0) \quad \text{for } |b - b_0| < \Delta b$$

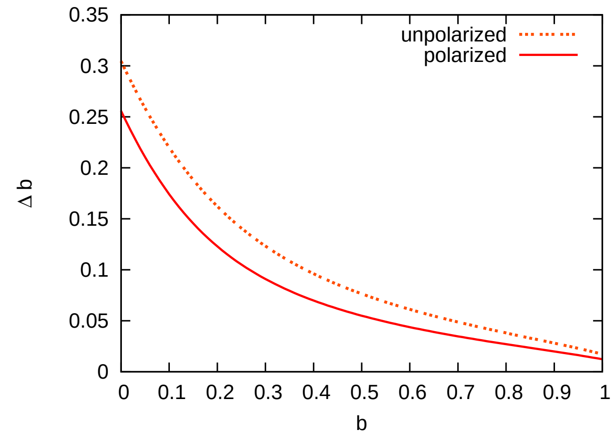
- Apply to the observables σ and P_t, using the fact that at a luminosity L,

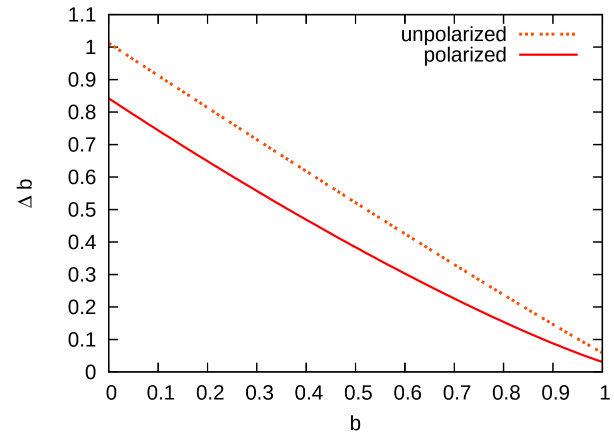
$$\Delta\sigma = f\sqrt{\frac{\sigma}{L}}\ , \qquad \Delta P_t = \frac{f}{\sqrt{\sigma L}}\sqrt{1 - P_t^2}$$

at a confidence level f (assuming no systematic error).

- For cross section and polarization asymmetry measurements respectively,

- Good possibility to distinguish between $b = 1$ and $b = 0$. b^2 dependence decreases the sensitivity. Things directly proportional to b?





All such studies look for difference caused by the different tensor structure to kinematical distributions.

Effect caused by CP mixing in MSSM, on the other hand, will only affect normalisations mostly.

Case of almost degenerate H/A needs to be discussed separately.Zerwas, Kalinowski, Choi,Pilaftsis, Lee..

CP-violating observables: constructed for ILC Hagiwara et al, Han et al, Biswal et al,Keung et al, Osland et al...

For the LHC: for example, R.G, Miller, Muehlleitner: hep-ph:0708.0458

For the PLC Hagiwara et al, Singh et al, Krawczyk et al.

Construct variables such that each probes one part of the anomalous coupling; thus CP violating variables to probe CP mixing.

- Cross-sections integrated over $CP\tilde{T}$ symmetric phase space will probe only the $CP - even$, $\tilde{T}$–even couplings, in the approximation that the anomalous couplings are small.

- Partially integrated cross-sections will be able to probe these. for example to probe a P-odd cpupling we construct Forward-Backward asymemtry.

- Constructed different observables out of the available momenta such that they have specific CP and $\tilde{T}$ transformation properties.

- Look at expectaion value of 'sign' of these observables.These asymemtries, are proportional to the part of the anomalous coupling which has the **same** CP and $\tilde{T}$ transformation properties as the observable, to leading order in the anomalous coupling.

- The up-down asymmetry of the $\bar{t}$ production w.r.t. the e^- - t plane ($\phi'_4 = 0$) is given by

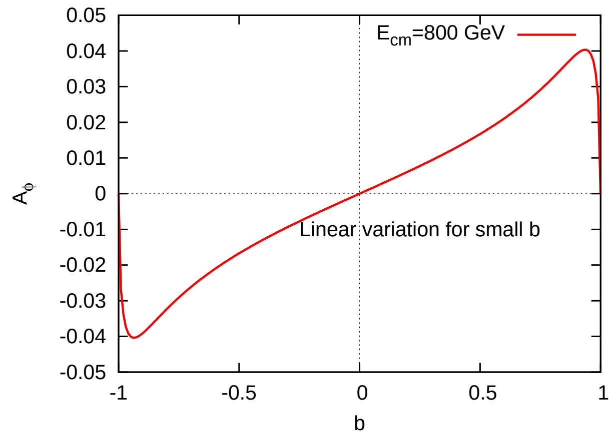
$$A_\phi = \frac{\sigma_{\text{partial}}(0 \leq \phi'_4 < \pi) - \sigma_{\text{partial}}(\pi \leq \phi'_4 < 2\pi)}{\sigma_{\text{partial}}(0 \leq \phi'_4 < \pi) + \sigma_{\text{partial}}(\pi \leq \phi'_4 < 2\pi)},$$

$$\text{with } \sin\phi'_4 = \frac{\vec{P} \cdot (\vec{p}_3 \times \vec{p'}_4)}{|\vec{P}| \cdot |\vec{p}_3 \times \vec{p'}_4|} \qquad (\vec{P} \equiv \vec{p}_1 - \vec{p}_2)$$

$\vec{p}'_4$ is the $\bar{t}$ momentum in the $\bar{t}$ - Higgs rest-frame.

- In terms of a and b, this asymmetry has the structure

$$A_\phi = \frac{x_\phi \; ab}{x_t - y_t \; b^2} = c x_\phi \; ab \sigma_{\text{tot}}$$



HZZ can be probed at LHC (e.g. R.G., Miller, Muhelleitner: hep-ph/0708.0458). But for lighter Higgs and/or HWW e^+e^- better?

Biswal,Singh, Choudhury and R.G. (changed notation!)

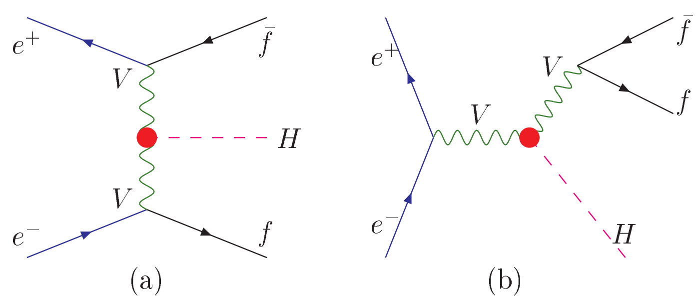
$$\Gamma_{\mu\nu} = g_V(a_V\ g_{\mu\nu} + \frac{b_V}{m_V^2}(k_{1\nu}k_{2\mu} - g_{\mu\nu}\ k_1 \cdot k_2) + \frac{\tilde{b_V}}{m_V^2}\ \epsilon_{\mu\nu\alpha\beta}\ k_1^\alpha k_2^\beta)$$

Trans.	a_V	$\Re(b_V)$	$\Im(b_V)$	$\Re(\tilde{b}_V)$	$\Im(\tilde{b}_V)$
CP	+	+	+	−	−
$\tilde{T}$	+	+	−	−	+

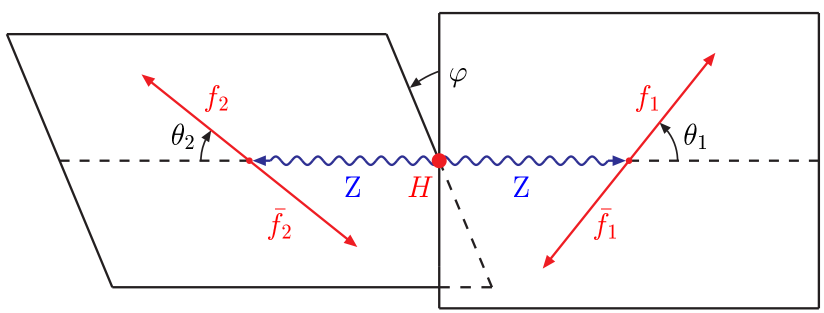
We construct observables with specific transformation properties under CP and $\tilde{T}$ to probe the anomalous coupling with corresponding property.

Use

$e^+ \, e^- \to f \; \bar{f} \; V^* \; V^* \to f \; \bar{f} \; H$ $e^+ \, e^- \to Z^* \to Z \; H \to f \; \bar{f} \; H$ with $H \to b\bar{b}$



Recall definition of the polar angles θ_i $(i = 1, 2)$ and the azimuthal angle φ for the sequential decay $H \to Z^{(*)}Z \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ in the rest frame of the Higgs boson.



Need to distinguish between f_1 and $\bar{f}_1$.
One Z decays to $f_1\bar{f}_1$ and other two $f_2\bar{f}_2$.

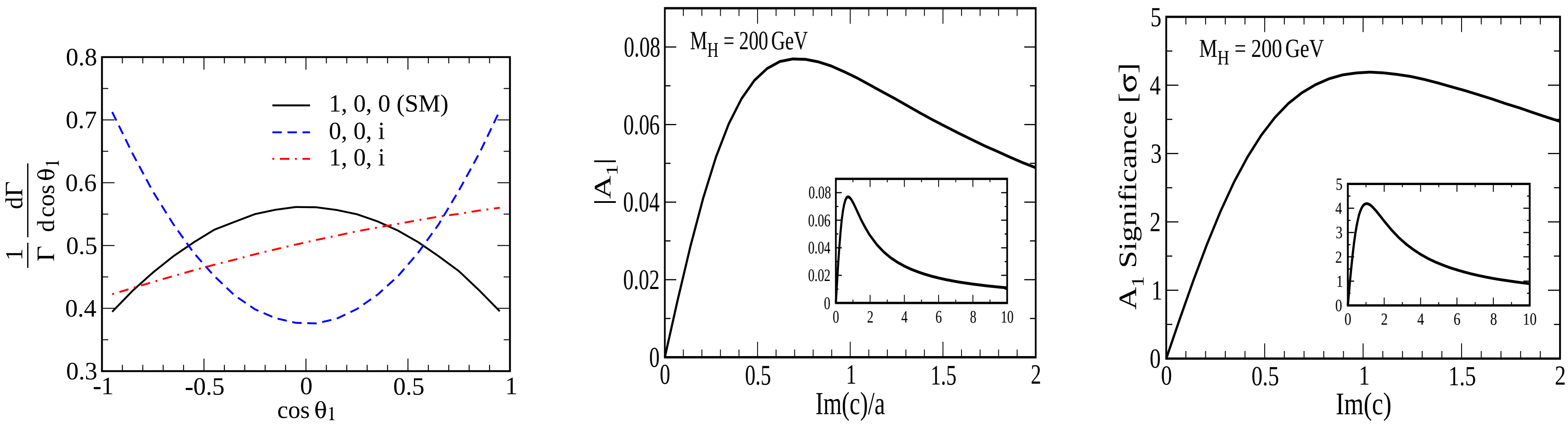
With these angles consruct different observabels:

$$O_1 \equiv \cos\theta_1 = \frac{(\vec{p}_{\bar{f}_1} - \vec{p}_{f_1}) \cdot (\vec{p}_{\bar{f}_2} + \vec{p}_{f_2})}{|\vec{p}_{\bar{f}_1} - \vec{p}_{f_1}||\vec{p}_{\bar{f}_2} + \vec{p}_{f_2}|}$$

$a \equiv a_Z, b \equiv b_Z, c \equiv \tilde{b}_Z$.

$$\mathcal{A}_1 = \frac{\Gamma(\cos\theta_1 > 0) - \Gamma(\cos\theta_1 < 0)}{\Gamma(\cos\theta_1 > 0) + \Gamma(\cos\theta_1 < 0)}.$$

If $\Im m(c) \neq 0$ this will mean $\mathcal{A}_1 \neq 0$.



The normalized differential width for $H \to ZZ \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ The solid (black) curve: the SM ($a = 1$, $b = c = 0$), Dashed (blue) curve: pure CP-odd state ($a = b = 0$, $c = i$). The dot-dashed (red) curve is for a state with a CP violating coupling ($a = 1$, $b = 0$, $c = i$). One can clearly see an asymmetry about $\cos\theta_1 = 0$ for the CP violating case.

Corrected for change in the production rate due to our non-standard couplings as compared to the SM rate. For $100 fb^{-1}$. **Calculated for LHC.**

May be improved by using jets instead of f_2 as the asymmetry does not require charge determination. One essentially means 'b'-jets. ATLAS study demonstrates it is possible to see the signal in $Z \to b\bar{b}$.

$CP = +$ and $\tilde{T} = -$

Couplings : $\Im(b_V)$

Observable : $A_f^{com} = \frac{\sigma(\mathcal{C}_{+-}>0)-\sigma(\mathcal{C}_{+-}<0)}{\sigma(\mathcal{C}_{+-}>0)+\sigma(\mathcal{C}_{+-}<0)} = \frac{(F'U)+(B'D)-(F'D)-(B'U)}{(F'U)+(B'D)+(F'D)+(B'U)}$

where,

$$\mathcal{C}_{+-} = \left[(\vec{p}_{e^-} - \vec{p}_{e^+}) \cdot \vec{p}_H\right] \left[\left[(\vec{p}_{e^-} - \vec{p}_{e^+}) \times \vec{p}_H\right] \cdot (\vec{p}_f - \vec{p}_{\bar{f}})\right]$$

$F' \equiv \cos\theta_H > 0$, $B' \equiv \cos\theta_H < 0$

$U \equiv \sin\phi_f > 0$, $D \equiv \sin\phi_f < 0$ $\mathcal{C}_{+-} > 0 \equiv (F'U) + (B'D)$

Combined asymmetry

$CP = -$ and $\tilde{T} = -$

Couplings : $\Re(\tilde{b}_V)$

Observable : $A_{UD}(\phi_f) = \frac{\sigma(\mathcal{C}_{--}>0)-\sigma(\mathcal{C}_{--}<0)}{\sigma(\mathcal{C}_{--}>0)+\sigma(\mathcal{C}_{--}<0)} = \frac{\sigma_U-\sigma_D}{\sigma_U+\sigma_D}$ where,

$$\mathcal{C}_{--} = \left[\left[(\vec{p}_{e^-} - \vec{p}_{e^+}) \times \vec{p}_H\right] \cdot (\vec{p}_f - \vec{p}_{\bar{f}})\right]$$

$\sigma_U \equiv \sigma(\sin\phi_f > 0)$, $\sigma_D \equiv \sigma(\sin\phi_f < 0)$

$\mathcal{C}_{--} > 0 \equiv \sigma_U$: Upward-f.

Up-down asymmetry

Needs Charge measurement of f !!

$L = 500$ fb^{-1}, at 3σ significance;

Coupling		Limit	Observable used
$\|\Delta a_Z\|$	$\leq$	0.034	σ with $R2$ cut; $f = e^-$
$\|\Re(b_Z)\|$	$\leq$	0.0044	σ with $R1$ cut; $f = \mu, q$
$\|\Im(b_Z)\|$	$\leq$	0.14	A^{com} with $R1$ cut; $f = \mu^-, e^-$
$\|\Re(\tilde{b}_Z)\|$	$\leq$	0.057	$A_{UD}(\phi_{e^-})$ with $R2$ cut
$\|\Im(\tilde{b}_Z)\|$	$\leq$	0.038	$A_{FB}(c_H)$ with $R1$ cut; $f = \mu, q$

For $f = b$, i.e. $Z \to b\bar{b}$ with charge measurement of b-quarks above limits can be improved to :

20% efficiency : $|\Im(b_Z)| \leq 0.050$, $|\Re(\tilde{b}_Z)| \leq 0.049$

30% efficiency : $|\Im(b_Z)| \leq 0.041$, $|\Re(\tilde{b}_Z)| \leq 0.046$

♢ $e^+e^- \to t\bar{t}\Phi$ offers the best probe of CP violation in the Higgs sector. The energy dependence and the polarisaiton of the top quark can be utilised.

♢ For a light higgs, $m_\phi \sim 120$ GeV the e^+e^- colliders offer much better prospect of probing the anomalous ΦVV coupling.

♢ ILC and $\gamma\gamma$(M. Krawczyk's talk) colliders provide the best bet to measure the CP mixing in the Higgs sector!