Simulating NNLO QCD corrections for processes with giant K factors

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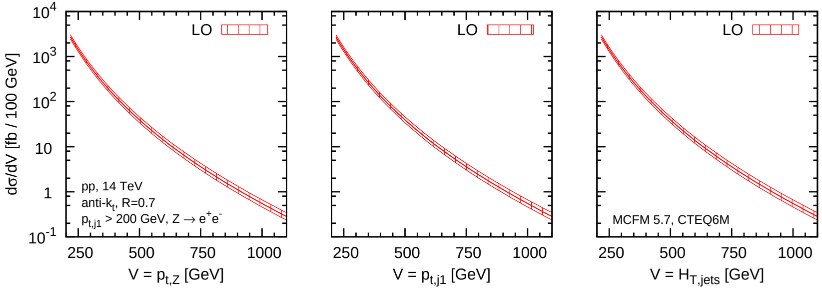
HP^2.3rd, Florence, 14-17 September 2010

[1] M.Rubin, G.P.Salam and SS, arXiv:1006.2144 [hep-ph]

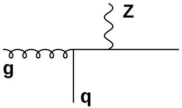
The problem of giant K factors

- Z+j at the LHC

$\mathbf{p_{t,Z}}$ $\mathbf{p_{t,\text{hardest jet}}}$ $\mathbf{H_{T,jets}} = \sum_{\text{all jets}} p_{t,j}$



LO:



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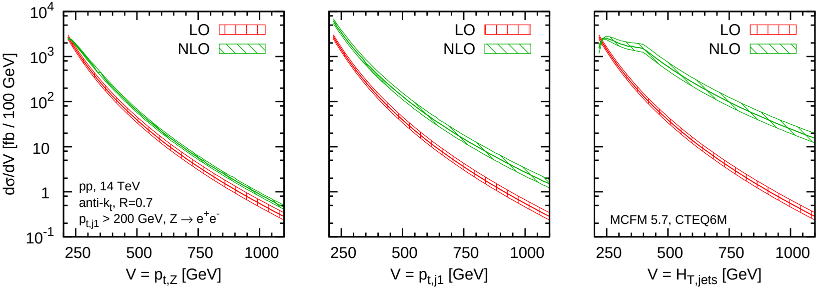
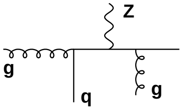
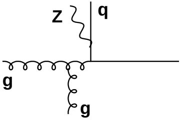


$\mathcal{O}\left(\alpha_{\rm EW}\alpha_s^2\right)$ | $\mathcal{O}\left(\alpha_{\rm EW}\alpha_s^2 \ln^2 p_{t,j1}/M_Z\right)$

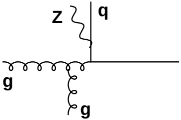
NLO:

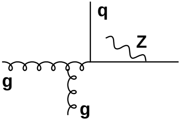




What do we have and what is missing?

- The large K factor for the Z+jet comes from the new "dijet type" topologies that appear at NLO





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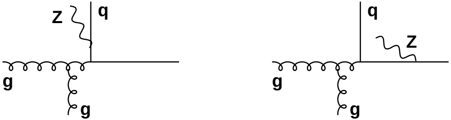

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- this raises doubts about the accuracy of these predictions
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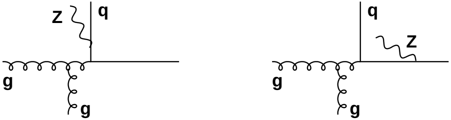

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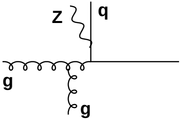

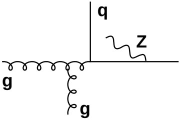

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- **2-loop part**
  - we need it to cancel IR and collinear divergences from Z+2j at NLO result
  - it will have the topology of Z+j at LO so it will not contribute much to the cross sections with giant K-factor

The basic idea

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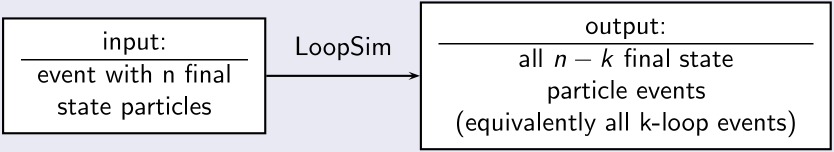
LoopSim procedure



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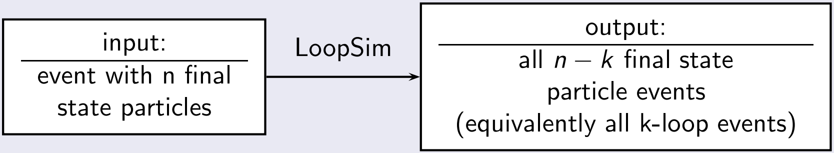

- notation:

$\bar{n}$**LO**	–	simulated 1-loop
$\bar{n}\bar{n}$**LO**	–	simulated 2-loop and simulated 1-loop
$\bar{n}$**NLO**	–	simulated 2-loop and exact 1-loop

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input: event with n final state particles	—— LoopSim ——>	output: all $n-k$ final state particle events (equivalently all k-loop events)

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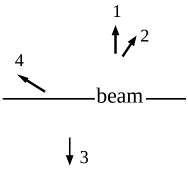
- this will work very well for the processes with large K factors e.g.

$$\sigma_{\bar{n}\mathrm{NLO}} = \sigma_{\mathrm{NNLO}}\left(1 + \mathcal{O}\left(\frac{\alpha_s^2}{K_{\mathrm{NNLO}}}\right)\right), \quad K_{\mathrm{NNLO}} \gtrsim K_{\mathrm{NLO}} \gg 1$$

The LoopSim method: $\bar{n}$LO, $\bar{n}\bar{n}$LO etc.

Input event



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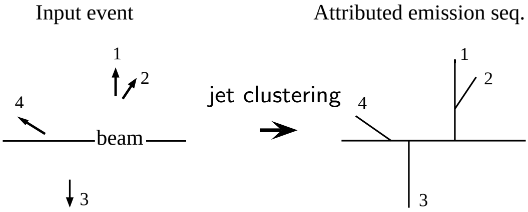

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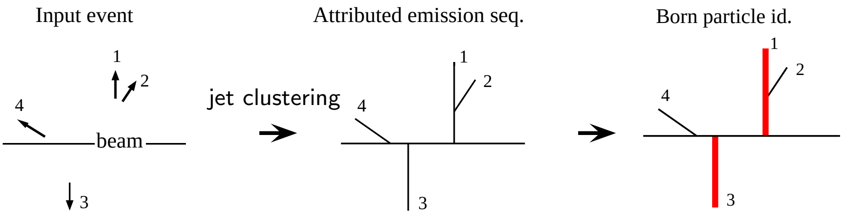

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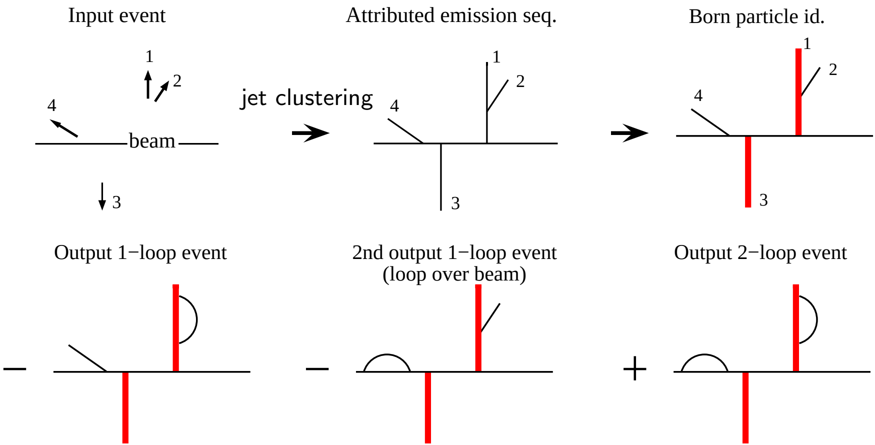

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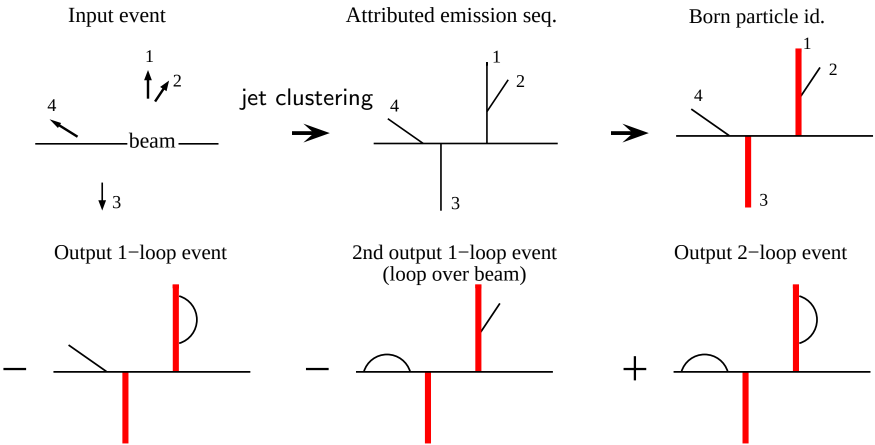

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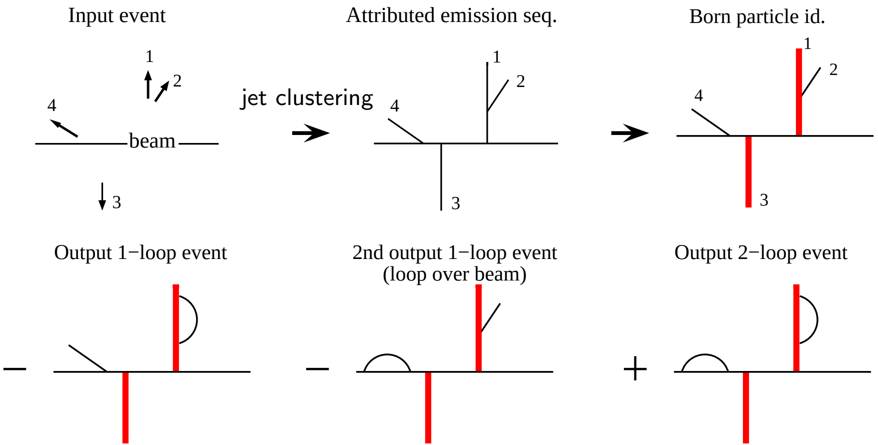

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- beware: the loops above are just a shortcut notation!

Including exact loops

$E_{n,l}$	–	input event with n final state particles and l loops
U_l^b	–	operator producing event with b Born particles and l loops
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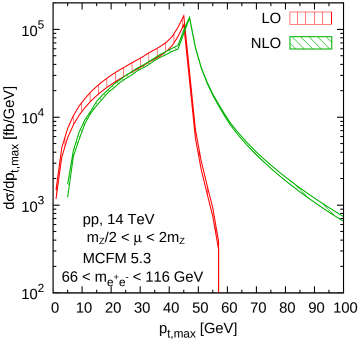
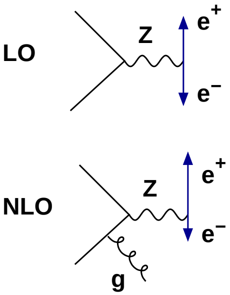
- inclusion of exact loops helps reducing scale uncertainties
- straightforward generalization to arbitrary number of exact loops

Validation

Drell-Yan at NNLO: spectrum of harder lepton





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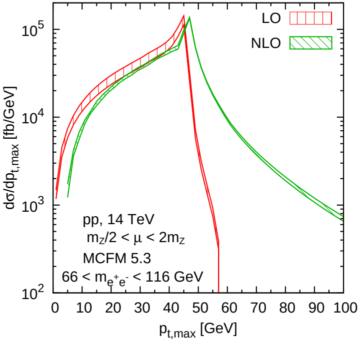

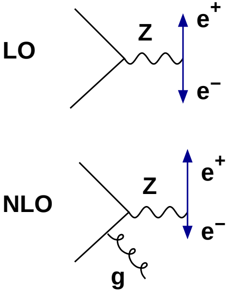

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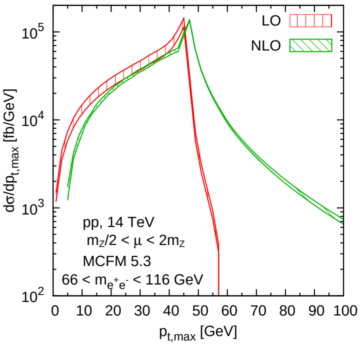

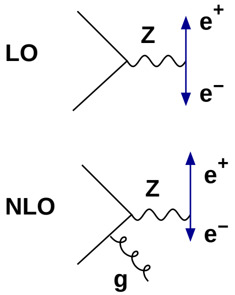

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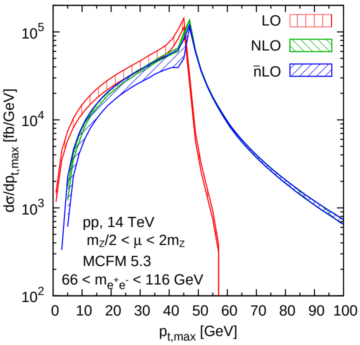

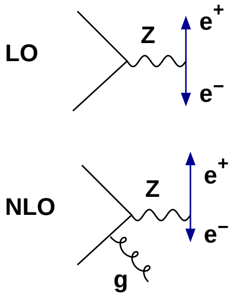

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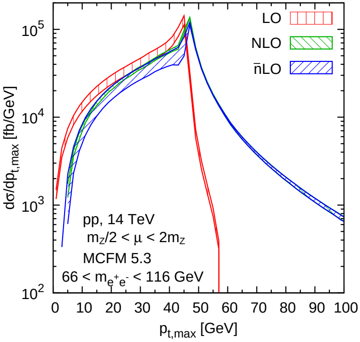

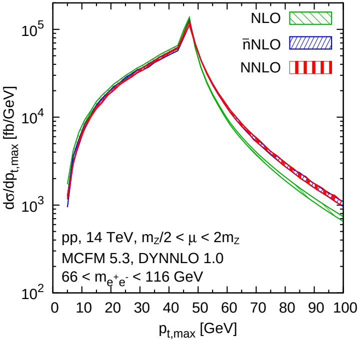

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$\bar{n}$NLO predictions for LHC

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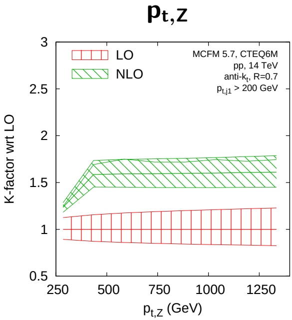

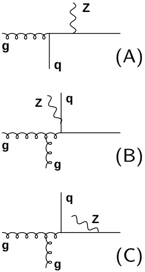

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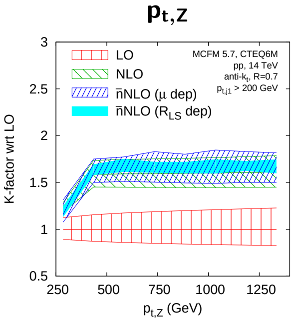

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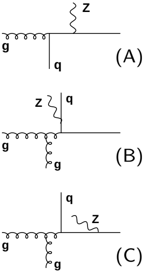

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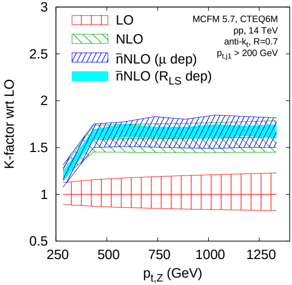
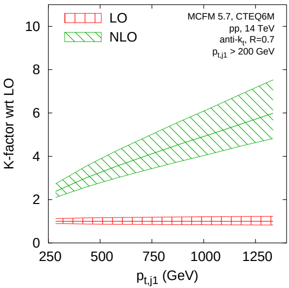


$p_{t,\text{hardest jet}}$



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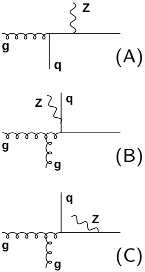

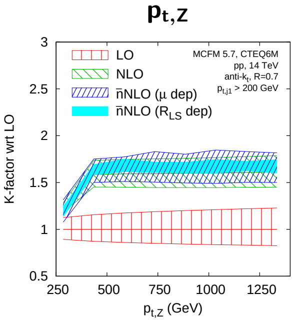

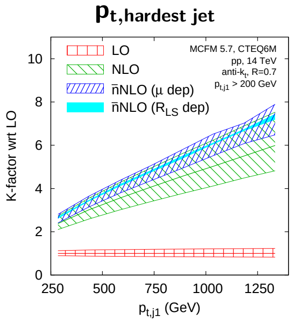

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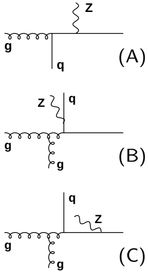

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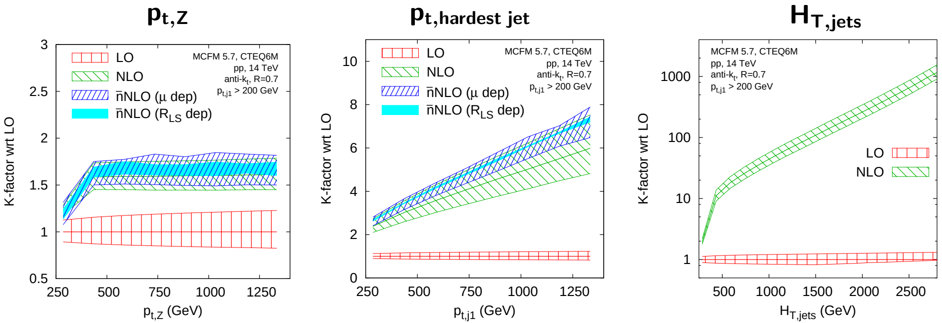

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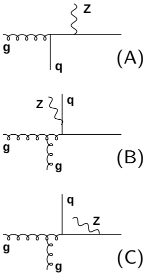

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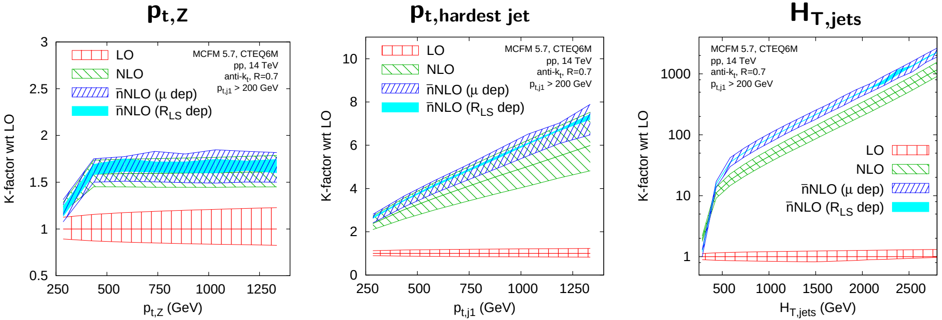

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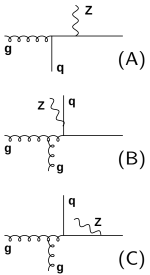
- $H_{T,\text{ jets}}$: significant correction; K factor ~ 2; given that it is more like going from LO to NLO this may happen sometimes, especially for nontrivial observables like H_T; can we understand it here?



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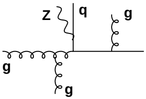

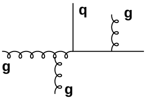

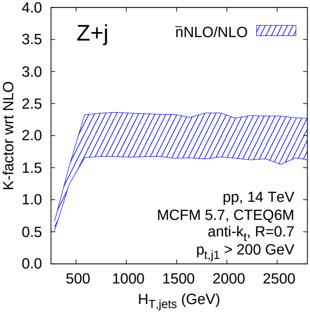

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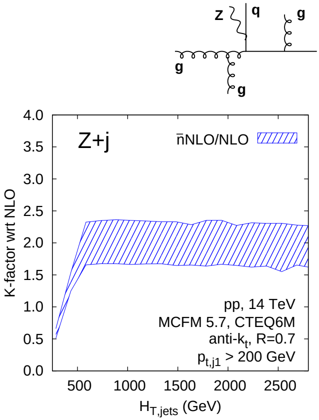

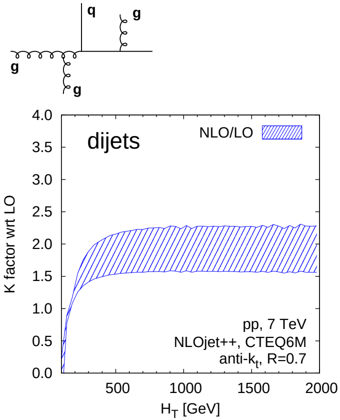

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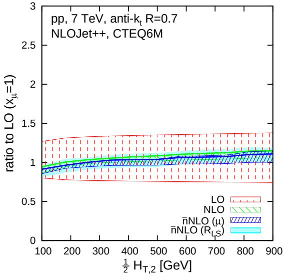

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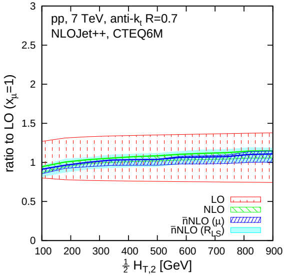

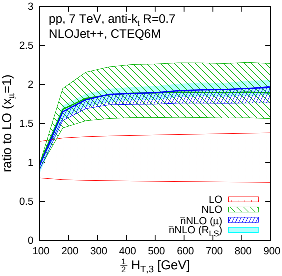

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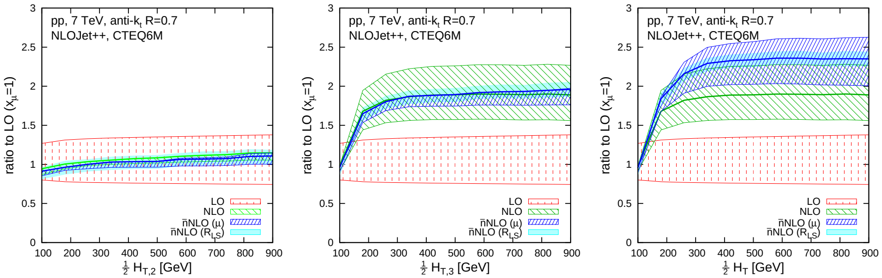

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- **$H_{T,3}$ converges, significant reduction of scale uncertainty:** the observable comes under control at $\bar{n}$NLO
- **H_T does not converge:** again caused by the initial state radiation, this time a second emission which shifts the distribution of H_T to higher values and causes no effect for the $H_{T,3}$ distribution

Summary

- several cases of observables with giant NLO K factor exist
- those large corrections arise due to appearance of new topologies at NLO
- we developed a method, called *LoopSim*, which allows one to obtain approximate NNLO corrections for such processes
- the method is based on unitarity and makes use of combining NLO results for different multiplicities
- we gave arguments why the method should produce meaningful results and we validated it against NNLO Drell-Yan and also NLO Z+j and NLO dijets
- we computed approximated NNLO corrections to Z+j and dijets at the LHC finding, depending on observable, either indication of convergence of the perturbative series or further corrections
- the latter has been understood and attributed to the initial state radiation

Outlook

- processes with W, multibosons, heavy quarks, ...

BACKUP SLIDES

The LoopSim method: some more details

For a given input E_n event with n final state particles the weights of all diagrams generated by LoopSim sum up to zero (unitarity)

$$\sum_{\text{all diagrams}} w_n = \sum_{\ell=0}^{\upsilon} (-1)^{\ell} \binom{\upsilon}{\ell} = 0\,, \qquad \ell - \text{number of loops, } \upsilon - \text{maximal } \ell$$

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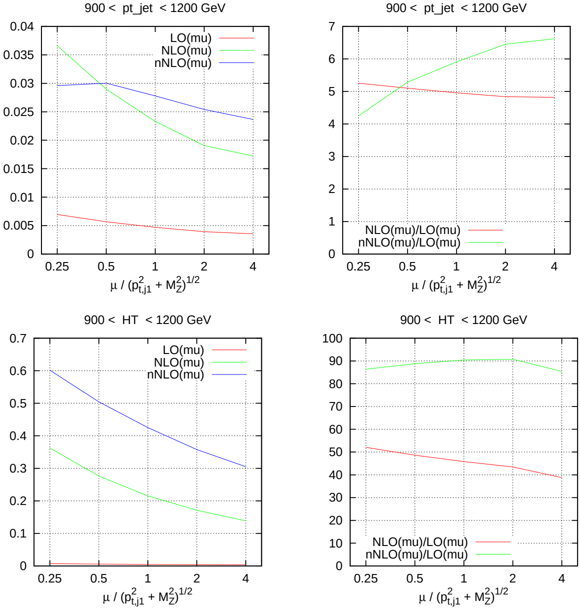
The principle of the method is simple. There is, however, a number of issues that need to be addressed to fully specify the procedure and make it usable:

- infrared and collinear safety
- conservation of four-momentum
- choice of jet definition (algorithm, value of R)
- treatment of flavour (e.g. for processes with vector bosons)
  - Z boson can be emitted only from quarks and never itself emits
- extension to input events with exact loops

Scale dependence: Z + jet



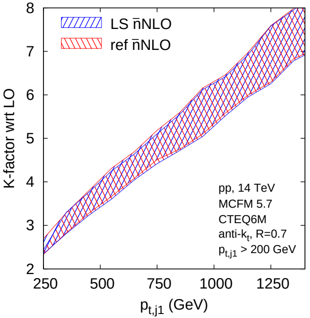
Reference-observable method

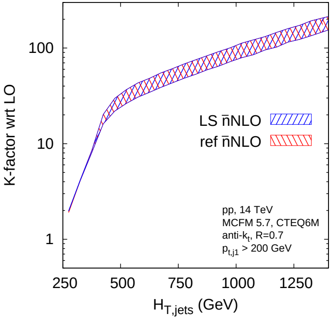
Take a reference observable identical at LO to the observable A

$$\sigma^{(A)}_{\text{Z+j@NNLO}} = \sigma^{(\text{ref})}_{\text{Z+j@NNLO}} + \left(\sigma^{(A)} - \sigma^{(\text{ref})}\right)_{\text{Z+j@NNLO}}$$
$$= \sigma^{(\text{ref})}_{\text{Z+j@NNLO}} + \left(\sigma^{(A)} - \sigma^{(\text{ref})}\right)_{\text{Z+2j@NLO}}$$

If the reference observable converges well

$$\sigma^{(A)}_{\text{Z+j@NNLO}} \simeq \sigma^{(\text{ref})}_{\text{Z+j@NLO}} + \left(\sigma^{(A)} - \sigma^{(\text{ref})}\right)_{\text{Z+2j@NLO}}$$





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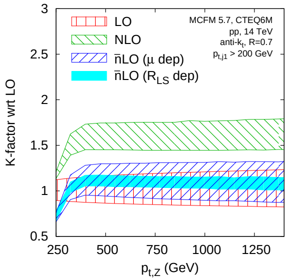

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  - correctly reproduced dip towards $p_t = 200$ GeV

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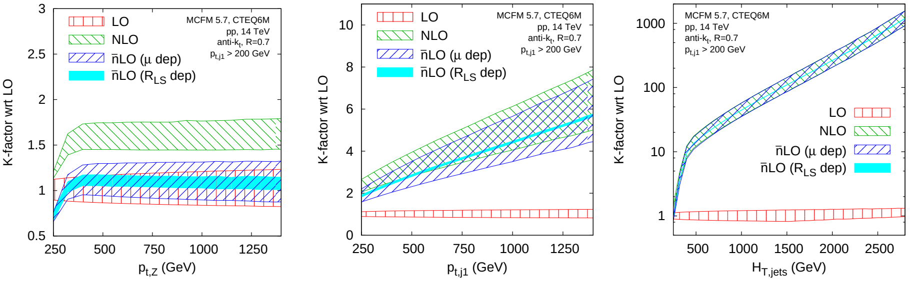

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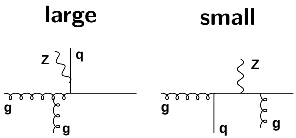

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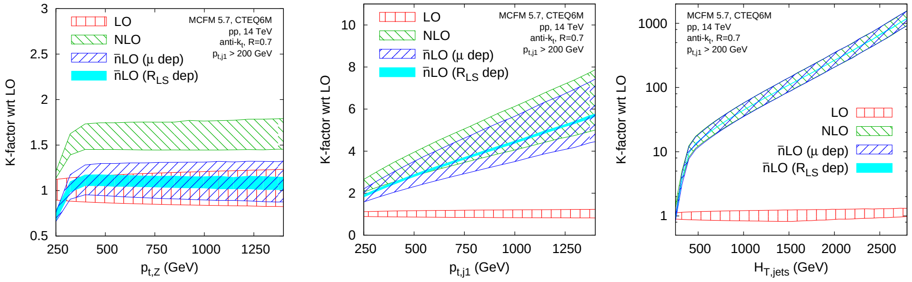

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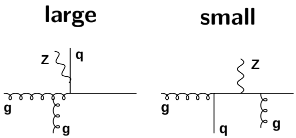
- small R uncertainties – driven only by subleading diagrams



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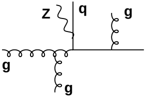

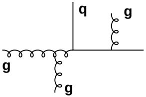

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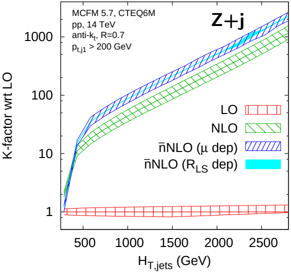

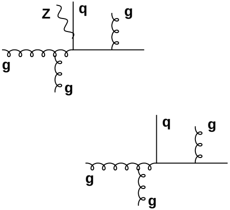

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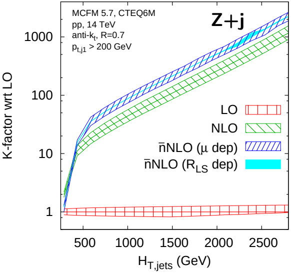

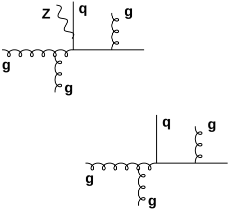

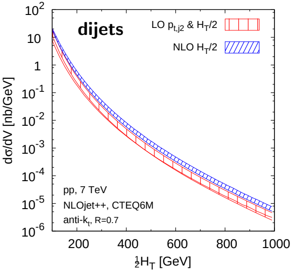

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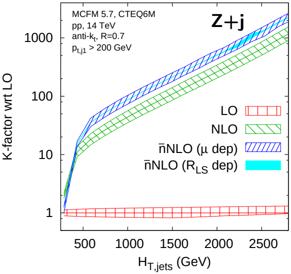

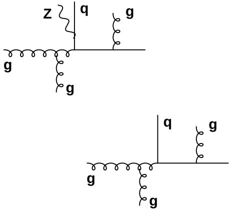

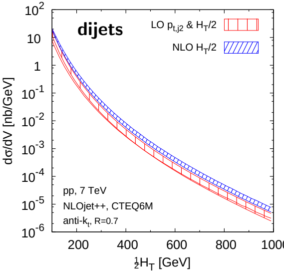

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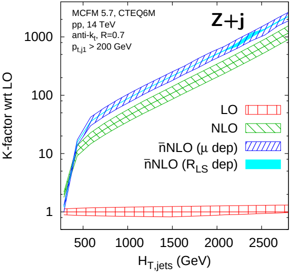

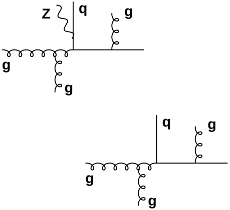

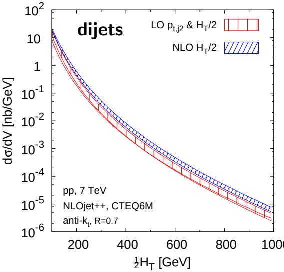

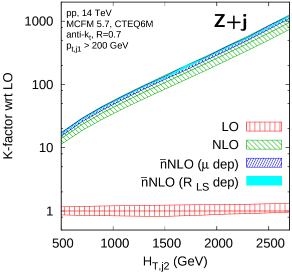

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  - and indeed it is small!