Integrable Quantum Circuits from Statistical Mechanics

arXiv.2206.15142

& work in preparation

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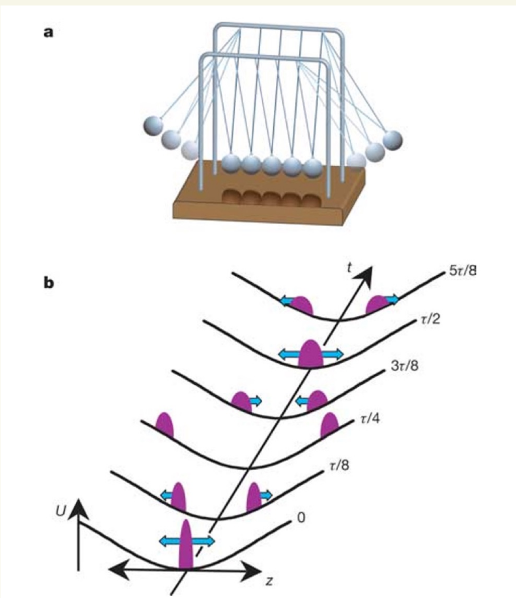
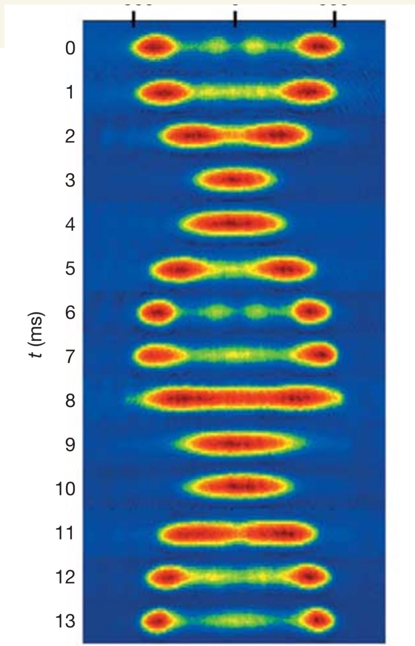
Why Non-Equilibrium

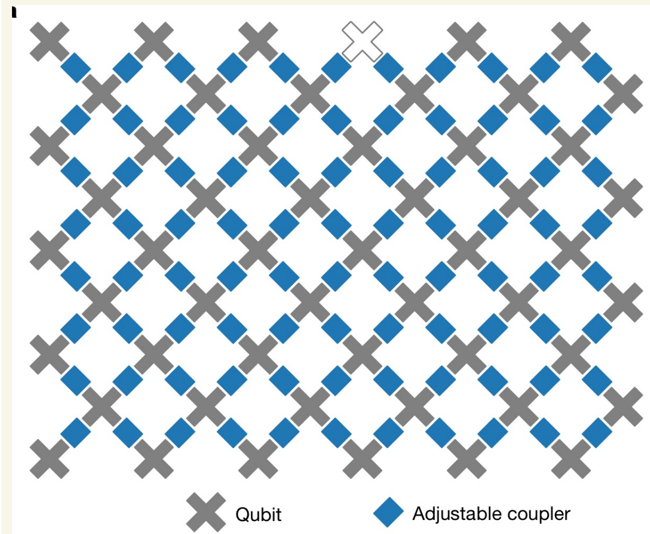
Out-of-equilibrium

discrete space-time





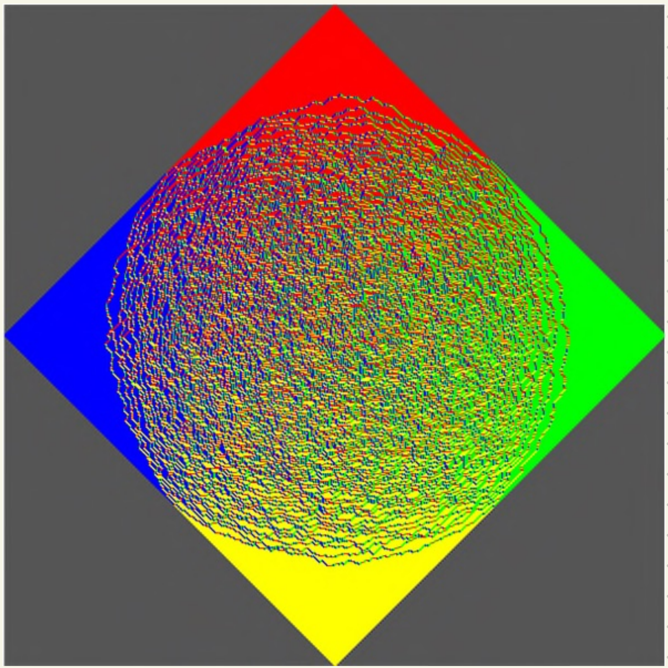
Kinoshita et al, 2006



Google team, 2019

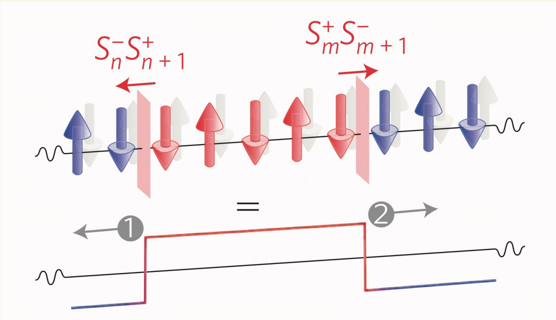
Exactly Solvable Models

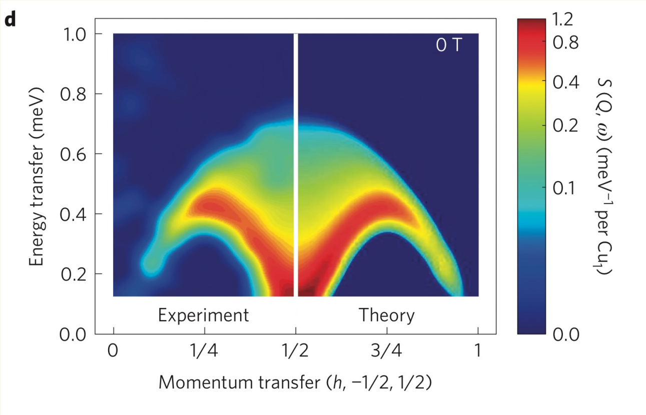
In stat-mech



⇒ exact ⇐
results
(sometimes
analytical)
even experiments!

In q. spin chain





Outline

* Quantum circuits as Floquet system
* Yang-Baxter integrability in a nutshell
* "Floquet Baxterisation"
* Conclusion & Outlook

Floquet Circuits

$$H = \sum_{m=1}^{L} h_{m,m+1} = H_1 + H_2$$

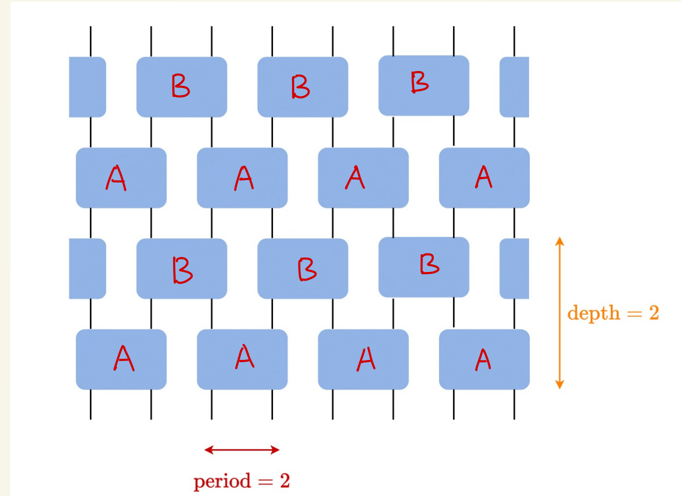
$$H_1 = \sum_m h_{2m-1,2m} \qquad H_2 = \sum_m h_{2m,2m+1}$$

time dependent Hamiltonian

$$H(t+T) = H(t) \Rightarrow U(nT) = \left[\mathcal{P}\exp\left(-i\int_0^T d\tau H(\tau)\right)\right]^n$$

$$U_F(T) = \exp(-iH_1T)\exp(-iH_2T)$$

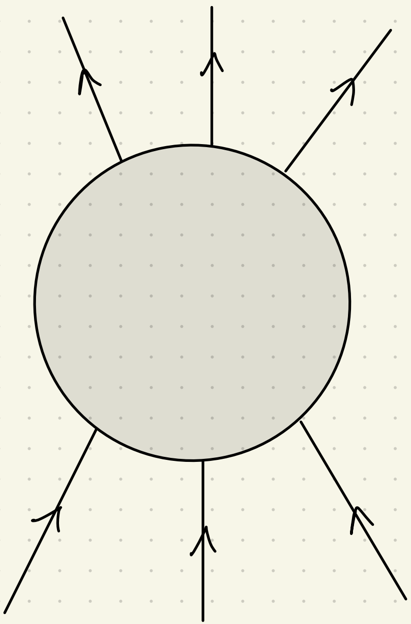
$$= \prod_{m=1}^{L/2} \underbrace{A_{2m-1,2m}}_{\exp(-ih_{2m-1,2m}T)} \prod_{m=1}^{L/2} \underbrace{B_{2m,2m+1}}_{\exp(-ih_{2m,2m+1}T)}$$

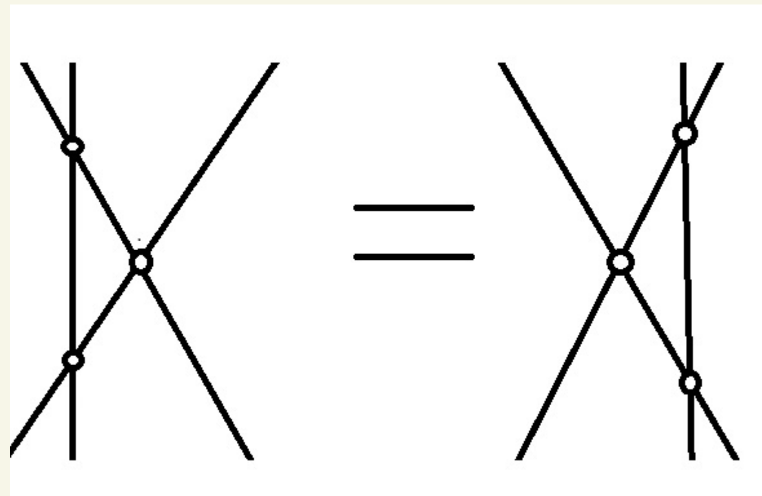


Quantum Circuits

$$\tilde{H}(t) = \begin{cases} H_1 & 0 \leq t < T \\ H_2 & T \leq t < 2T \end{cases}$$

Yang-Baxter integrability





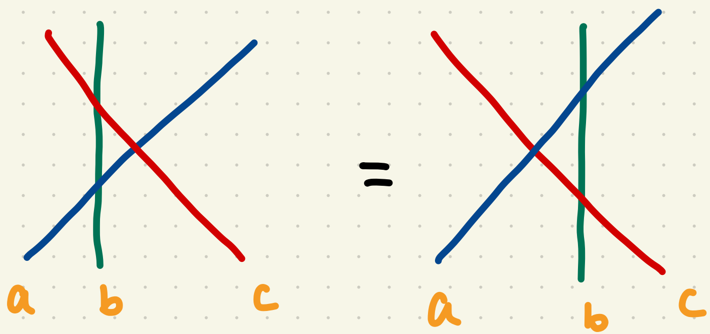
3-body scattering $\longrightarrow$ factorised scattering

Yang-Baxter Integrability

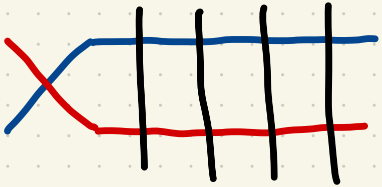
Hilbert space $(\mathbb{C}^N)^{\otimes L}$ "spins"

$R_{ab} : \mathbb{C}^N \otimes \mathbb{C}^N \to \mathbb{C}^N \otimes \mathbb{C}^N$

$$\Rightarrow \boxed{\begin{aligned} &R_{ab}(u,v)\, R_{ac}(u,w)\, R_{bc}(v,w) \\ &\quad = R_{bc}(v,w)\, R_{ac}(u,w)\, R_{ab}(u,v), \quad u,v,w \in \mathbb{C} \end{aligned}}$$



$\Rightarrow$ vertex model in stat-mech



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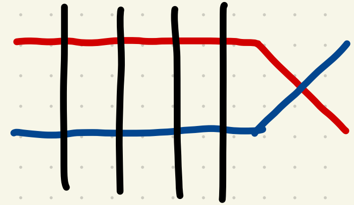
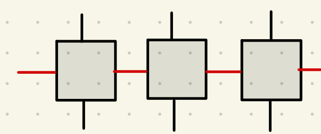


$$M_a(u,\{\xi_m\}) = \prod_{m=1}^{L} R_{a,m}(u,\xi_m) \Rightarrow R_{a,b}(u,v)\,M_a(u)\,M_b(v)$$
$$= M_b(v)\,M_a(u)\,R_{a,b}(u,v)$$

$$T(u) = \mathrm{Tr}_a\big(M_a(u)\big) \Rightarrow \boxed{[T(u), T(v)] = 0}$$

$T(u) =$

$\Updownarrow$



$$Q_n = -i\,\partial_u^{(n-1)} \log T(u)\Big|_{u=0} \Rightarrow \text{"local charges"}$$

Permutation op. $P_{a,b} : \mathbb{C}^N \otimes \mathbb{C}^N \to \mathbb{C}^N \otimes \mathbb{C}^N$

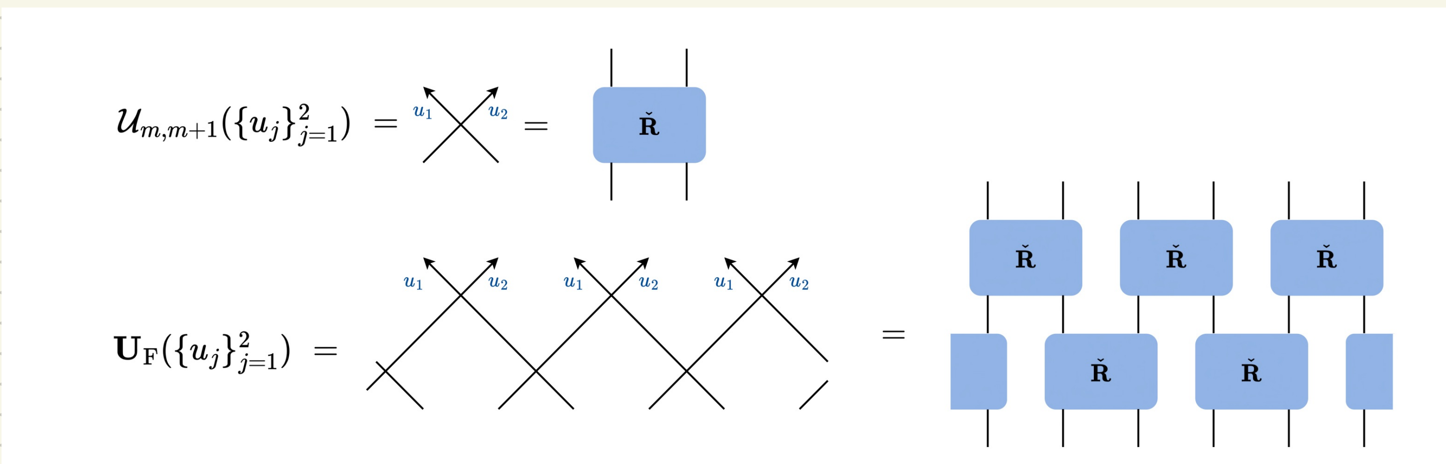
$$P_{a,b}\,\mathcal{O}_a = \mathcal{O}_b\,P_{a,b}$$

Floquet Baxterisation

$\xi_{2m-1} = 0$, $\xi_{2m} = \alpha$, $L \bmod 2 = 0$ $\quad \check{R}_{a,b}(u,v) = R_{a,b}(u,v) P_{a,b}$

Theorem.

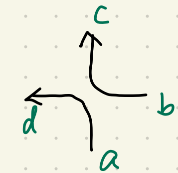
$$\boxed{U_F(\alpha) = \prod_{m=1}^{L/2} \check{R}_{2m-1,2m}(0,\alpha) \prod_{m=1}^{L/2} \check{R}_{2m,2m+1}(0,\alpha)\ , \quad [U_F(\alpha), T(u,\{\xi_m\})] = 0,\ \forall u \in \mathbb{C}}$$



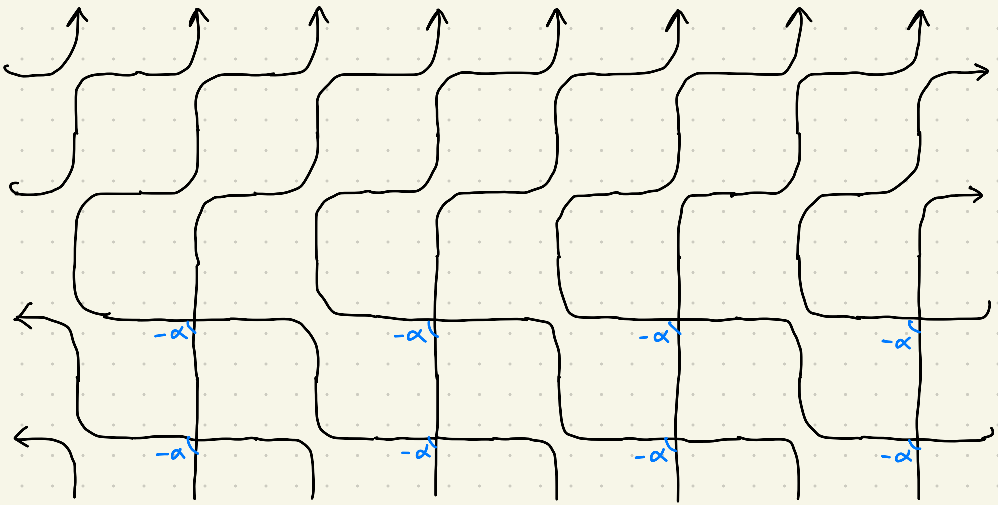
A graphic "proof":

permutation:

$$P^{ab}_{cd} = \delta^a_d \, \delta^b_c =$$



$$T^2(0,\{0,\alpha\})\, G^2 =$$



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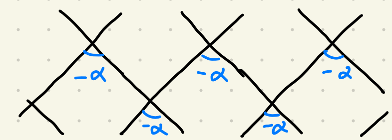

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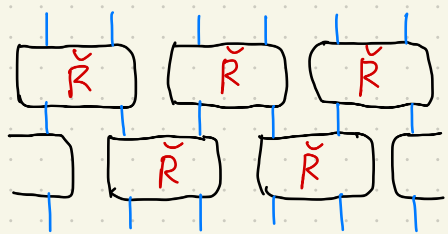


identical to q. circuit!

$\simeq$ "2 copies of transfer mat."

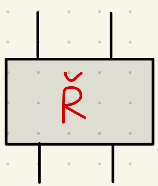
Example : 6-vertex model

$$\check{R}_{m,m+1}(u) = \begin{pmatrix} \sinh(u+\eta) & 0 & 0 & 0 \\ 0 & e^{u}\sinh\eta & \sinh u & 0 \\ 0 & \sinh u & e^{-u}\sinh\eta & 0 \\ 0 & 0 & 0 & \sinh(u+\eta) \end{pmatrix} \propto \exp(-i\, \underbrace{h_{m,m+1}}_{\text{XXZ hamiltonian}}\, t)$$

XXZ hamiltonian
($\Delta = \cosh\eta$)

$$h_{m,m+1} = \cosh\eta - \left(\sigma^x_m \sigma^x_{m+1} + \sigma^y_m \sigma^y_{m+1} + \underbrace{\cosh\eta}_{\Delta}\, \sigma^z_m \sigma^z_{m+1}\right) - \underbrace{\sinh\eta}\, \left(\sigma^z_m - \sigma^z_{m+1}\right)$$

if $\eta \in i\mathbb{R}$, $\sinh\eta \in i\mathbb{R}$
$(|\Delta| < 1)$



$\Rightarrow$ NOT UNITARY
when $|\Delta| < 1$

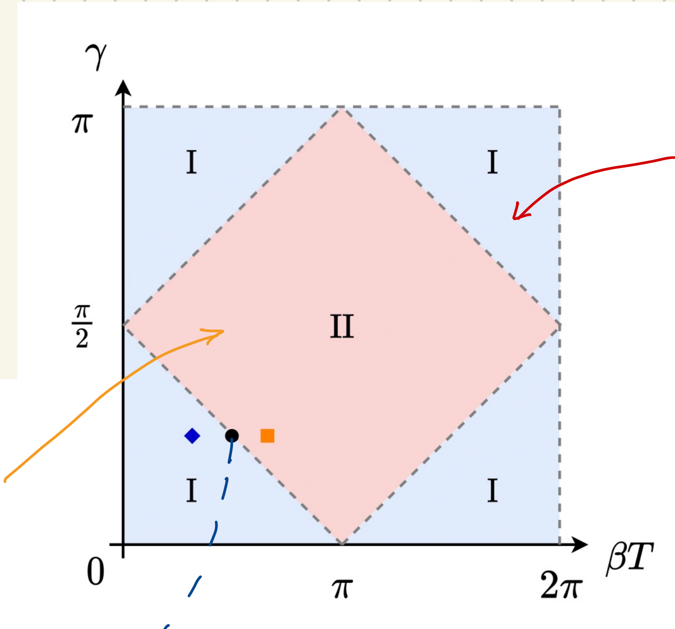
What about the spectrum of U_F ?

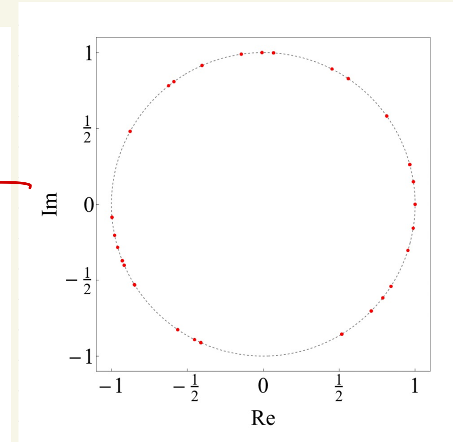
"Phase Transition"

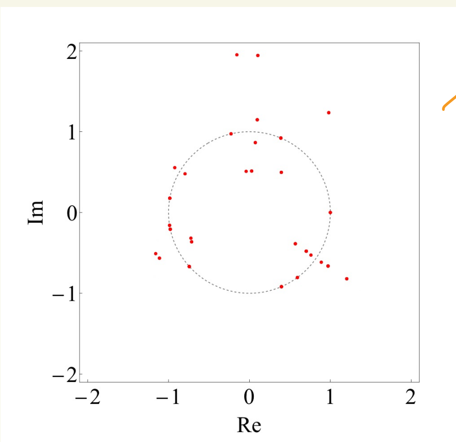
$$\gamma = \frac{\eta}{i}$$

$$\Delta = \cos\gamma$$

$$|\Delta| < 1$$







anti-unitary
symm. breaking

* works for finite size
* phase I : "pseudo-unitary"

Conclusion & Outlook

* Generic way of constructing integrable circuits
  "Floquet Baxterisation"

* Anti-unitary symm. breaking in 6-vertex circuits

Extention : open boundary , disordered , "n-qudit gate",

other circuits (star-triangle relation) , ...