Exactly solved models of many-body quantum chaos

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SLOVENIAN RESEARCH AGENCY

GGI Tea Break Seminar
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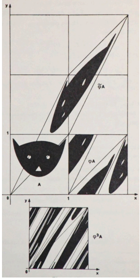
Goal: Find 'Baker and Cat maps' of many body quantum physics!

1. A proof of random-matrix spectral form factor
   `PRL` **`121`**`, 264101 (2018); CMP` **`387`**`, 597 (2021)`

2. Exact local dynamical correlation functions in **dual-unitary** models:
   An example of exact ergodic hierarchy of quantum many-body dynamics
   `PRL` **`123`**`, 210601 (2019),`

3. Dynamical complexity (entanglement entropy `PRX` **`9`**`, 021033 (2019)`, operator entropy `SciPost Phys.` **`8`**`, 067 (2020)`), and structural / perturbative stability of quantum ergodicity `PRX` **`11`**`, 011022 (2021)`.

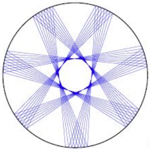


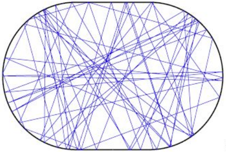
The Quantum Chaos Conjecture (aka BGS conjecture)

Casati, Guarnerri, Valz-Gris 1980, Berry 1981, Bohigas, Giannoni, Schmidt 1984

The spectral fluctuations of quantum systems with chaotic and ergodic classical limit are *universal* and described by Random Matrix Theory (RMT).

The same holds for periodically-driven systems if one instead considers the statistics of quasi-energies.


(a)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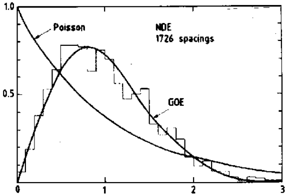

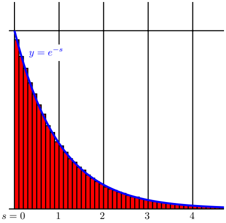

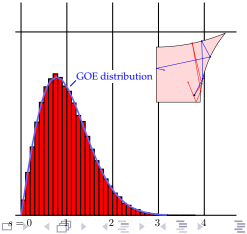

The **spectrum** $\{\varphi_n\}$ of a unitary one-period propagator $U = \mathcal{T}\exp(-i\int_0^1 H(t)\mathrm{d}t)$ as a **gas** in one dimension

Spectral density:

$$\rho(\varphi) = \frac{2\pi}{\mathcal{N}}\sum_n \delta(\varphi - \varphi_n).$$

Spectral pair correlation function (2-point function):

$$r(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi \rho(\varphi + \tfrac{1}{2}\vartheta)\rho(\varphi - \tfrac{1}{2}\vartheta) - 1.$$

Spectral Form Factor (SFF) (Fourier transform of 2-point function):

$$\begin{aligned} K(t) &= \frac{\mathcal{N}^2}{2\pi}\int_0^{2\pi} \mathrm{d}\vartheta r(\vartheta) e^{it\vartheta} = \sum_{m,n} e^{it(\varphi_m - \varphi_n)} - \mathcal{N}^2\delta_{t,0} \\ &= \left|\mathrm{tr}\, U^t\right|^2 - \mathcal{N}^2\delta_{t,0}, \quad t \in \mathbb{Z}. \end{aligned}$$

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Caveat: SFF is not self-averaging! Consider instead $\bar{K}(t) = \mathbb{E}[K(t)]$.

RMT (No time reversal symmetry):

$$K_{\rm CUE}(t) = t, \quad t < \mathcal{N}.$$

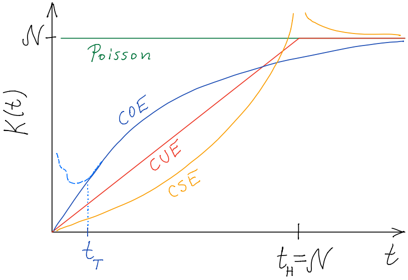
RMT (With time teversal symmetry):

$$K_{\rm COE}(t) = 2t - t\log(1 + 2t/\mathcal{N}), \quad t < \mathcal{N}.$$

Random (uncorrelated, Poissonian) spectrum $\{\varphi_n\}$:

$$K_{\rm Poisson} \equiv \mathcal{N}.$$

RMT vs Real System:



$$\mathbb{E}[K(t)] = \mathbb{E}\left[\sum_{m,n} e^{i(\varphi_m - \varphi_n)}\right].$$

Saturation $\bar{K}(t) \sim \mathcal{N}$ beyond Heisenberg time $t > t_{\rm H} = \mathcal{N} = 1/\Delta\varphi$.

Non-universal (system-specific) behaviour below Ehrenfest/Thouless time $t < t_{\rm T}$.

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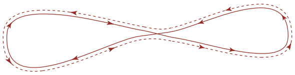

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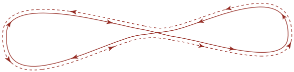
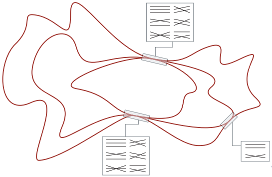


To all orders, RMT terms is reproduced by considering full combinatorics of self-encountering orbits [Müller et al, PRL 2004]



What about QCC for many-body systems at '$\hbar \sim 1$'? (say for interacting spin 1/2 or fermionic systems)

Disclaimer: This talk is not about 'large-N' QFTs, nor small-$\hbar$ many-body systems, so no saddle points, no Lyapunov chaos..

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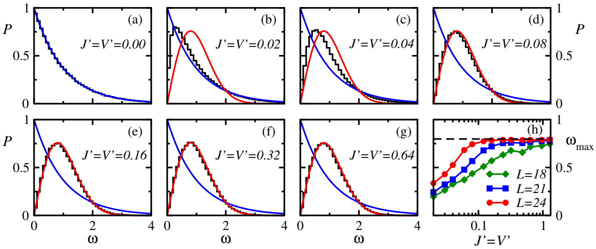
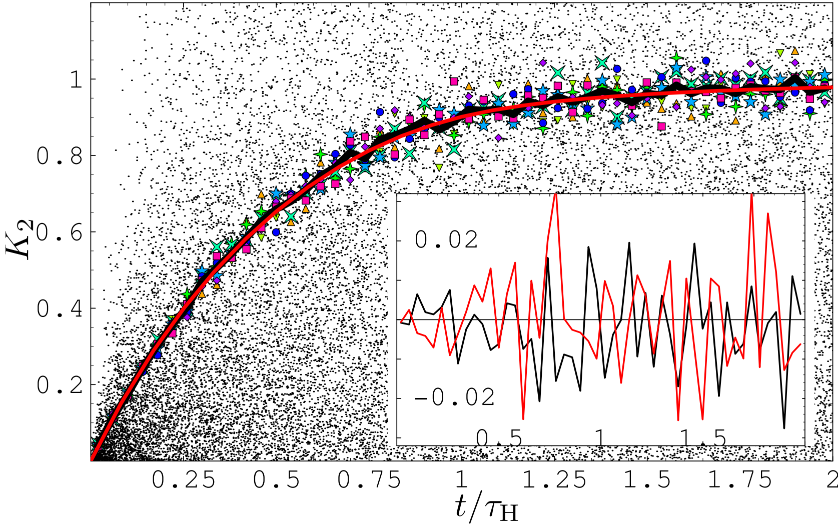
..Instead, it is about the models like:

$$H = \sum_{j=0}^{L-1} (-J c_j^\dagger c_{j+1} - J' c_j^\dagger c_{j+2} + \text{h.c.} + V n_j n_{j+1} + V' n_j n_{j+2}), \quad n_j = c_j^\dagger c_j.$$



From [Rigol and Santos, 2010]...numerical evidence since early 1990's

Clean non-integrable Kicked Ising Chain [Pineda and TP, PRE 2007]



Only recently first analytic results arrived..

Floquet long-ranged (non-integrable/non-mean field) spin 1/2 chains `[arXiv:1712.02665]`

PHYSICAL REVIEW X **8**, 021062 (2018)

Many-Body Quantum Chaos: Analytic Connection to Random Matrix Theory

Pavel Kos, Marko Ljubotina, and Tomaž Prosen*
Physics Department, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

(Received 5 February 2018; revised manuscript received 12 April 2018; published 8 June 2018)

Floqeut local quantum circuits with random unitary gates in the limit of large local Hilbert space dimension $q \to \infty$ `[PRL` **121**`, 060601 (2018); PRX` **8**`, 041019 (2018)]`

Solution of a minimal model for many-body quantum chaos

Amos Chan, Andrea De Luca and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom
(Dated: December 20, 2017)

Spectral statistics in spatially extended chaotic quantum many-body systems

Amos Chan, Andrea De Luca and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom
(Dated: April 4, 2018)

What about fermionic or spin 1/2 systems with strictly local interactions?

$$H_{\rm KI}[\boldsymbol{h};t] = H_{\rm I}[\boldsymbol{h}] + \delta_p(t) H_{\rm K}, \quad H_{\rm I}[\boldsymbol{h}] \equiv \sum_{j=1}^{L} \left\{ J\sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z \right\}, \;\; H_{\rm K} \equiv b \sum_{j=1}^{L} \sigma_j^x,$$

with Floquet propagator

$$U_{\rm KI} = e^{-iH_{\rm K}} e^{-iH_{\rm I}}.$$

J, b: homogeneous spin-coupling and transverse field
h_j position dependent longitudinal field

Kicked Ising model [TP, JPA 1998; PTPS 2000; PRE 2002]

$$H_{\mathrm{KI}}[\boldsymbol{h};t] = H_{\mathrm{I}}[\boldsymbol{h}] + \delta_p(t) H_{\mathrm{K}}, \quad H_{\mathrm{I}}[\boldsymbol{h}] \equiv \sum_{j=1}^{L} \left\{ J\sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z \right\}, \;\; H_{\mathrm{K}} \equiv b \sum_{j=1}^{L} \sigma_j^x,$$

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$$U_{\mathrm{KI}} = e^{-iH_{\mathrm{K}}} e^{-iH_{\mathrm{I}}}.$$

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Remarks:

- KI model is integrable if $b = 0$ or $h_j \equiv 0$.
- For generic h_j and $b \neq 0$, the model has no symmetries.

The disorder averaging

$$H_{\mathrm{KI}}[\boldsymbol{h};t] = H_{\mathrm{I}}[\boldsymbol{h}] + \delta_p(t) H_{\mathrm{K}}, \quad H_{\mathrm{I}}[\boldsymbol{h}] \equiv \sum_{j=1}^{L} \left\{ J\sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z \right\}, \quad H_{\mathrm{K}} \equiv b \sum_{j=1}^{L} \sigma_j^x,$$

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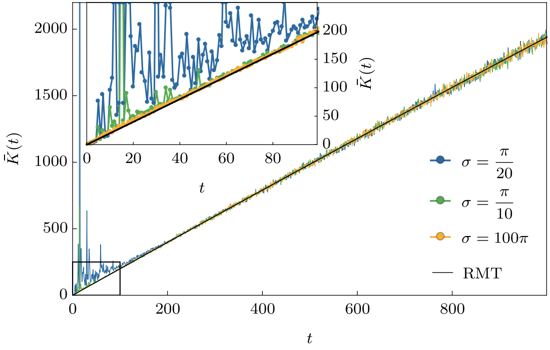
Consider longitudinal magnetic field h_j to be i.i.d. (Gaussian) variable

$$\bar{K}(t) = \mathbb{E}_{\boldsymbol{h}}[K(t)] = \int_{-\infty}^{\infty} \left(\prod_{j=1}^{L} \frac{\mathrm{d}h_j}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(h_j - \bar{h})^2}{2\sigma^2} \right) \right) K(t).$$



For $|J| = |b| = \pi/4$ and σ large enough the behaviour seems immediately RMT-like ($t_{\mathrm{T}} \sim 1$)

Interpreting $\bar{K}(t)$ in terms of a partition function of $2d$ classical statistical model, we can study SFF analytically in thermodynamic limit!

Exact SFF in thermodynamic limit [PRL **121**, 264101 (2018)]

Theorem: For *odd* t:

$$\lim_{L\to\infty} \bar{K}(t) = \begin{cases} 2t-1\,, & t \le 5 \\ 2t\,, & t \ge 7 \end{cases}.$$

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Conjecture: For *even* t:

$$\bar{K}(2) = 2,\ \bar{K}(4) = 7,\ \bar{K}(6) = 13,\ \bar{K}(8) = 18,\ \bar{K}(10) = 22,$$
$$\bar{K}(t) = 2t+1, \quad t \ge 12.$$

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Remarks:

- Results independent of $\sigma > 0$: The model is ergodic for any disorder strength (**no Floquet-MBL!**). In particular, we can take the limit of a clean system at the end $\sigma \searrow 0$.
- Results independent of $\bar{h}$: We can set $\bar{h} = 0$ which corresponds to a limiting integrable system.

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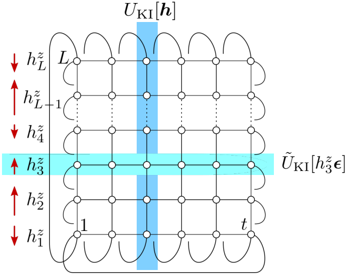
We found a simple locally interacting model with finite dimensional local Hilbert space with proven RMT spectral correlations at all time-scales!

The trace of U_{KI}^t is equivalent to a partition sum of a classical 2d Ising model with **row-homogeneous field** h_j:

$$\mathrm{tr}\,(U^t_{\mathrm{KI}}[\boldsymbol{h}]) \sim$$



Duality relation:

$$\mathrm{tr}\,\left(U_{\mathrm{KI}}[\boldsymbol{h}]\right)^t = \mathrm{tr}\left(\prod_{j=1}^{L} \tilde{U}_{\mathrm{KI}}[h_j\boldsymbol{\epsilon}]\right)$$

where $\boldsymbol{\epsilon} = (1, 1 \ldots, 1)$ and $\tilde{U}_{\mathrm{KI}}$ is a KI model on a ring of size t with twisted parameters $\tilde{J}(J, b)$, $\tilde{b}(J, b)$.

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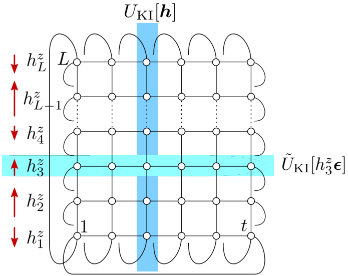

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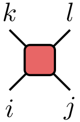
Remarkably: $\tilde{U}_{\mathrm{KI}}$ is **unitary** for $|J| = |b| = \pi/4$ (Self-dual, $J = \pm\tilde{J}, b = \pm\tilde{b}$)
Observed first in [Akila, Waltner, Gutkin and Guhr, JPA **49**, 375101 (2016)]

Consider a unitary gate on a two-qudit system $U \in U(d^2)$ and define the following duality transformation

$$\sim: U \longmapsto \tilde{U}\,,$$

via reshuffling of basis states

$$\langle j| \otimes \langle \ell|\tilde{U}|i\rangle \otimes |k\rangle = \langle k| \otimes \langle \ell|U|i\rangle \otimes |j\rangle\,.$$



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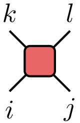


We call a gate dual-Unitary, if not only U is unitary, i.e.

$$UU^\dagger = U^\dagger U = \mathbb{1},$$

but also $\tilde{U}$ is unitary

$$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger \tilde{U} = \mathbb{1}.$$

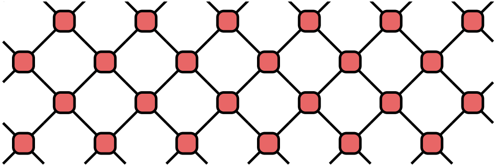
One step of a quantum circuit is a unitary over $(\mathbb{C}^d)^{\otimes 2L}$

$$\mathbb{U} = \mathbb{U}^{\mathrm{o}}\mathbb{U}^{\mathrm{e}}$$

where

$$\mathbb{U}^{\mathrm{e}} = U^{\otimes L}, \quad \mathbb{U}^{\mathrm{o}} = \Pi_{2L}\mathbb{U}^{\mathrm{e}}\Pi_{2L}^{\dagger}$$

and Π_ℓ is a periodic translation $\Pi_\ell |i_1\rangle \otimes |i_2\rangle \cdots |i_\ell\rangle \equiv |i_2\rangle \otimes |i_3\rangle \cdots |i_\ell\rangle \otimes |i_1\rangle$.



(here $t = 2$ and $L = 6$)

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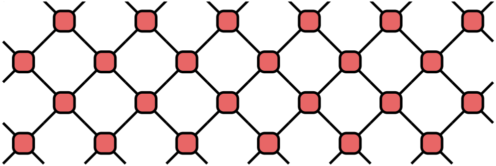

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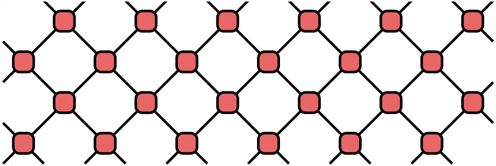

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Clearly we have duality of traces

$$\mathrm{tr}\,\mathbb{U}^t = \mathrm{tr}\,\tilde{\mathbb{U}}^L\,.$$

Problem: Classify all Dual Unitary gates for a given dimension d

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We can provide a complete classification only for $d = 2$:

$$U = e^{i\phi}(u_+ \otimes u_-) \cdot V[J] \cdot (v_- \otimes v_+)\,,$$

where $\phi, J \in \mathbb{R}$, $u_\pm, v_\pm \in \mathrm{SU}(2)$ and

$$V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right].$$

Relevant examples:

1. SWAP gate $U = V[\pi/4] = S$.
2. One parameter line of the trotterized XXZ chain

$$U_{\mathrm{XXZ}} = V[J]\,,$$

3. The maximally chaotic self-dual kicked Ising (SDKI) chain

$$U_{\mathrm{SDKI}} = e^{-ih\sigma^z} e^{i\frac{\pi}{4}\sigma^x} \otimes e^{i\frac{\pi}{4}\sigma^x} \cdot V[0] \cdot e^{i\frac{\pi}{4}\sigma^y} e^{-ih\sigma^z} \otimes e^{i\frac{\pi}{4}\sigma^y}.$$

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1. SWAP gate $U = V[\pi/4] = S$.
2. One parameter line of the trotterized XXZ chain

$$U_{\mathrm{XXZ}} = V[J]\,,$$

3. The maximally chaotic self-dual kicked Ising (SDKI) chain

$$U_{\mathrm{SDKI}} = e^{-ih\sigma^z} e^{i\frac{\pi}{4}\sigma^x} \otimes e^{i\frac{\pi}{4}\sigma^x} \cdot V[0] \cdot e^{i\frac{\pi}{4}\sigma^y} e^{-ih\sigma^z} \otimes e^{i\frac{\pi}{4}\sigma^y}.$$

Problem: Classify all Dual Unitary gates for a given dimension d

We can provide a complete classification only for $d = 2$:

$$U = e^{i\phi}(u_+ \otimes u_-) \cdot V[J] \cdot (v_- \otimes v_+)\,,$$

where $\phi, J \in \mathbb{R}$, $u_\pm, v_\pm \in \mathrm{SU}(2)$ and

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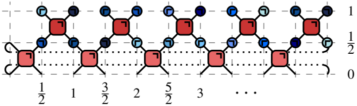
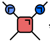
See [`Claeys & Lamacraft, PRL`**`126`**`, 100603 (2021)`] for generalization (not complete classification!) to higher d, and [`Gutkin,Braun,Akila,Waltner,Guhr, arXiv:2001.01298`] for generalization of KI model to higher d.

$\mathbb{U}_L =$

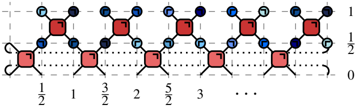


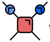
$U_{x+\frac{1}{2},\frac{1}{2}} = (u_x \otimes u_{x+\frac{1}{2}})\, U =$  ,

$U_{x,1} = (w_{\mathrm{mod}(x-\frac{1}{2},L)} \otimes w_x)\, W =$  ,

$$K(t,L) = \mathbb{E}_u[|\mathrm{tr}\,\mathbb{U}_L^t|^2] = \mathbb{E}_u[\mathrm{tr}\,(\mathbb{U}_L^\dagger \otimes \mathbb{U}_L^T)^t] = \mathrm{tr}[(\mathbb{E}_u[\tilde{\mathbb{U}}_t^\dagger \otimes \tilde{\mathbb{U}}_t^T])^L = \mathrm{tr}\,\mathbb{T}^L.$$

Spectral Form Factor in DUC

$\mathbb{U}_L =$ 

$U_{x+\frac{1}{2},\frac{1}{2}} = (u_x \otimes u_{x+\frac{1}{2}})\, U =$  ,

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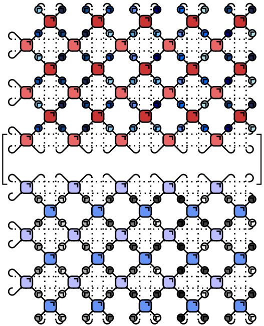
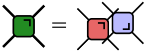
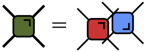
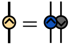
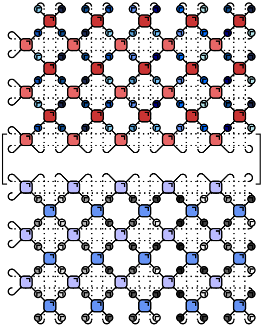
$K(t,L) = \mathbb{E}\left[\mathrm{tr}(\mathbb{U}_L)^t \mathrm{tr}(\mathbb{U}_L^\dagger)^t\right] = \mathbb{E}\Big[$  $\Big]$

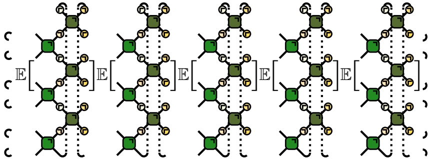
‘Thermofield-double’ (aka *folded*) representation of SFF

 $= U \otimes U^*$,  $= W \otimes W^*$,

 $= u_x \otimes u_x^*, \;\; w_x \otimes w_x^*.$

$$K(t,L) = \mathbb{E}\left[\mathrm{tr}(\mathbb{U}_L)^t \mathrm{tr}(\mathbb{U}_L^\dagger)^t\right] = \mathbb{E}$$



$=$ 

Theorem [Bertini, Kos, P, CMP 387, 597 (2021)], written for $d = 2$:

For i.i.d. local 1-qubit gates u_x, w_x with arbitrary *smooth* (and nonsingular) distribution over $SU(2)$, and for any dual unitary 2-qubit gates U other than the SWAP, we have

$$\lim_{L\to\infty} K(t) = \dim \{M_{a,\iota}, M_{ab,\iota}; a, b \in \{x, y, z\}, \iota \in \{0,1\}\}' = t$$

$$\sigma^{\alpha}_{\tau} = \mathbb{1}_{2\tau} \otimes \sigma^{\tau} \otimes \mathbb{1}_{2t-2\tau-1} \in \mathrm{End}((\mathbb{C}^2)^{\otimes 2t}), \quad \tau \in \frac{1}{2}\mathbb{Z}_{2t},$$

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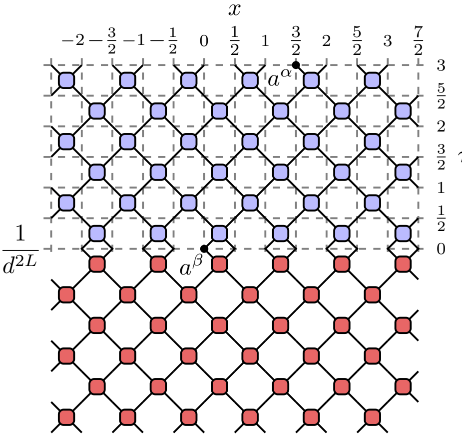
Clearly, the *minimal* set of generators of the commutant is spanned by t integer-site translation operators. The crux of the proof is to show that there is no other!

Writing the orthonormal set of local observables as a^α, $\alpha = 0, \ldots, d^2-1$, $\mathrm{tr}\left[(a^\alpha)^\dagger a^\beta\right] = d\,\delta_{\alpha,\beta}$ and choose $a^0 = \mathbb{1}$, so all other a^α are traceless, we shall be interested in the following space-time correlator

$$D^{\alpha\beta}(x,y,t) \equiv \frac{1}{d^{2L}}\mathrm{tr}\left[a^\alpha_x \mathbb{U}^{-t} a^\beta_y \mathbb{U}^t\right] = \begin{cases} C^{\alpha\beta}_-(x-y,t) & 2y \text{ even} \\ C^{\alpha\beta}_+(x-y,t) & 2y \text{ odd} \end{cases},$$



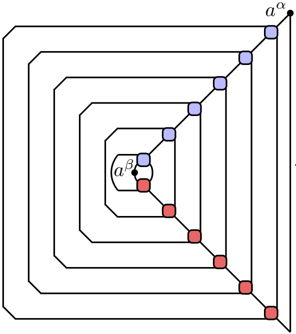
,

Property 1

If U is dual-unitary, the dynamical correlations are non-zero for $t \leq L/2$ only on the edges of a lightcone spreading at speed ± 1

$$C_\nu^{\alpha\beta}(x,t) = \delta_{x,\nu t} C_\nu^{\alpha\beta}(\nu t, t)\,, \qquad \nu = \pm,\ \alpha, \beta \neq 0\,.$$

$$C_+^{\alpha\beta}(t,t) = \frac{1}{d^{2t+1}}$$



.

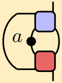
Property 2

The light cone correlations $C_{+}^{\alpha\beta}(t,t)$ and $C_{-}^{\alpha\beta}(-t,t)$ are given by

$$C_{\nu}^{\alpha\beta}(\nu t,t) = \frac{1}{d}\mathrm{tr}\left[\mathcal{M}_{\nu}^{2t}(a^{\beta})a^{\alpha}\right],$$

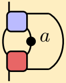
where we introduced the linear maps over $\mathrm{End}(\mathbb{C}^{d})$

$$\mathcal{M}_{+}(a) = \frac{1}{d}\mathrm{tr}_1\left[U^{\dagger}(a\otimes \mathbb{1})U\right] = \frac{1}{d}$$



,

$$\mathcal{M}_{-}(a) = \frac{1}{d}\mathrm{tr}_2\left[U^{\dagger}(\mathbb{1}\otimes a)U\right] = \frac{1}{d}$$



.

$\mathrm{tr}_i[A]$ denote partial traces over i-th site ($i = 1, 2$).

Dynamical correlations tamed!

Decay of correlations is given in terms of d^2-1 eigenvalues $\lambda_{+,\alpha}$ of single qudit channel ($d^2 \times d^2$ matrix) $\mathcal{M}_+$, and d^2-1 eigenvalues $\lambda_{-,\alpha}$ of $\mathcal{M}_-$.

$$D^{\alpha\beta}(x,y,t) = \begin{cases} \delta_{y-x,t} \sum_{\gamma=1}^{d^2-1} c^{\alpha\beta}_{-,\gamma} (\lambda_{-,\gamma})^{2t} & 2y \text{ even} \\ \delta_{x-y,t} \sum_{\gamma=1}^{d^2-1} c^{\alpha\beta}_{+,\gamma} (\lambda_{+,\gamma})^{2t} & 2y \text{ odd} \end{cases}$$

(One eigenvalue is always $\lambda_{\nu,0}=1$, with eigenoperator $a^0 = \mathbb{1}$.)

Classification of ergodic behaviours:

1. Non-interacting dynamics:
   all $\lambda_{\nu,\gamma}=1$ (example: SWAP $U|i\rangle \otimes |j\rangle = |j\rangle \otimes |i\rangle$)
2. Non-ergodic (and generically *non-integrable*) behavior:
   $\exists$ additional eigenvalue equal to one $\lambda_{\nu,\gamma}=1$, $\gamma \neq 0$.
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   all $\lambda_{\nu,\gamma\neq 0} \neq 1$, but $\exists \gamma \neq 0, |\lambda_{\nu,\gamma}|=1$.
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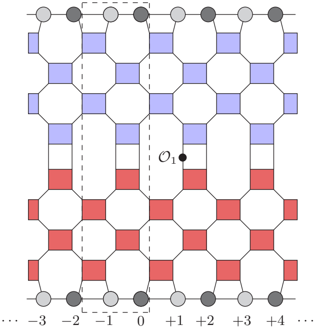
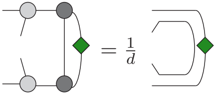
L. Piroli, B. Bertini, J. I. Cirac and TP, PRB **101**, 094304 (2020)

$$\lim_{L\to\infty} \langle\Psi|\mathbb{U}^{-t}\mathcal{O}_1\mathbb{U}^{t}|\Psi\rangle =$$



Exactly solvable staggered MPS initial states satisfying:

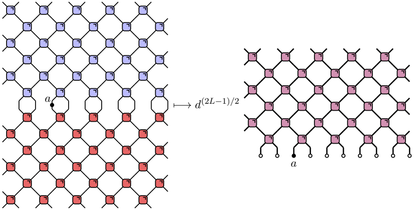


Operator entanglement in DUC

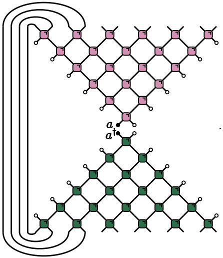
Analytic computation of Renyi-2 operator entanglement entropy for spreading of local operators [Bertini, Kos & P, SciPost Phys. 2020]:

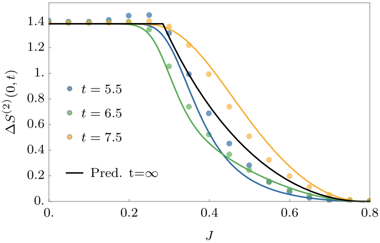
$$E_{op}(t) = \alpha t$$

where $\alpha = 2\log d$ signals maximal chaos.



$\rho_A(t, y; a) = \mathrm{tr}_{\bar{A}}[|a_y(t)\rangle\langle a_y(t)|] =$





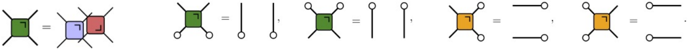
Recent results on space-time dual circuits beyond dual unitarity:

`Garratt, Chalker, PRX` **11**`, 021051 (2021); PRL` **127**`, 026802 (2021)`

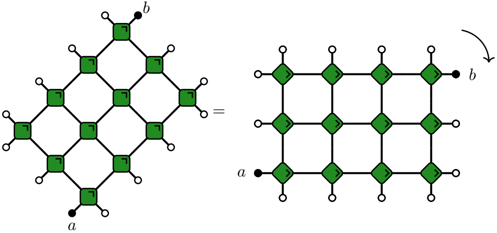
`Ippoliti, Khemani, PRL` **126**`, 060501 (2021)`
`Ippoliti, Rakovszky, Khemani, arXiv:2103.06873`
`Lu, Grover, arXiv:2103.06356`

`Lerose, Sonner, Abanin, PRX` **11**`, 021040 (2021)`
`Sonner, Lerose, Abanin, arXiv:2103.13741`

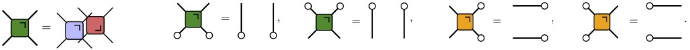
our group: `PRX` **11**`, 011022 (2021)`

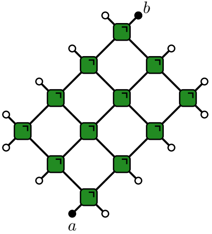
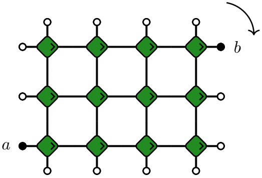


$\langle b_{x+y} | a_y(t) \rangle =$

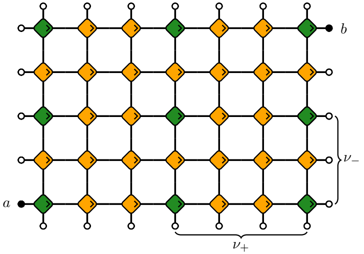
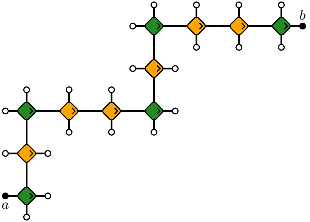


Structural (perturbative) stability of DUC [PRX 11, 011022 (2021)]



$\langle b_{x+y}|a_y(t)\rangle =$  $=$ 

$$U_\eta = U_{\mathrm{DU}} \cdot e^{i\eta P}$$

$\langle b_x|a_0(t)\rangle =$  $= \sum$ 

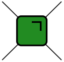
Reduced gates/circuits and dual-bistochastic Markov circuits

The U(1)-noise averaged dynamical correlations

$$c_{ab}(x,t) = \mathbb{E}_{\{h_{j,t}\}} C_{ab}(x,t), \quad U_{j,j+1} \to U_{j,j+1} e^{i h_{j,t} \sigma^z_j + i h_{j+1,t} \sigma^z_{j+1}}$$

can be formulated in terms of classical bistochastic brickwork Markov circuits in the basis of diagonal operators $|\mathbb{1}\rangle$, $|\sigma^z\rangle$ with elementary 2-gate

$$w :=$$



$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p\varepsilon & a & b \\ 0 & c & q\varepsilon & d \\ 0 & e & f & g \end{pmatrix},$$

$\varepsilon = 0$ corresponds to dual-unutary/dual-bistochastic circuit.

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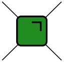
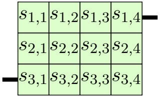
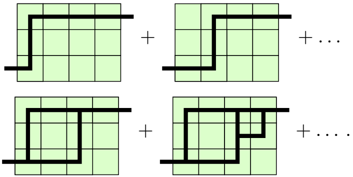
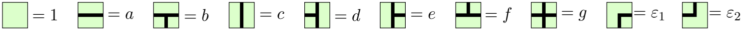
$w :=$  $= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p\varepsilon & a & b \\ 0 & c & q\varepsilon & d \\ 0 & e & f & g \end{pmatrix},$

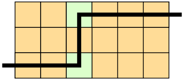
$\varepsilon = 0$ corresponds to dual-unutary/dual-bistochastic circuit.
Tilling representation of dynamical correlations ($\varepsilon_1 = p\varepsilon, \varepsilon_2 = q\varepsilon$):

$\langle \bullet_x | \bullet_0 (t) \rangle = \sum_{s_{ij} \in \text{tiles}}$ 

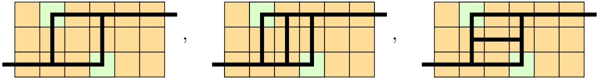
$=$ 



To fixed, say 2nd order in $\varepsilon_1, \varepsilon_2$, we get contributions from the no-loop (skeleton) diagram



as well as from higher, loop diagrams



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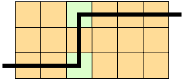

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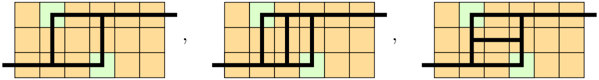


However, if

$$|a| > a^2 + \frac{|bf|}{1-\alpha}, \qquad \text{or} \qquad |c| > c^2 + \frac{|de|}{1-\beta}$$

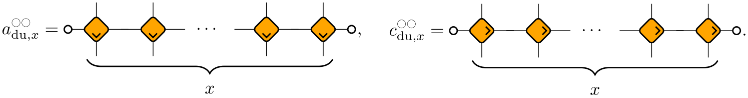
where α and β are, respectively, the largest singular values of

$$\begin{pmatrix} c & e \\ d & g \end{pmatrix}, \qquad \text{and} \qquad \begin{pmatrix} a & f \\ b & g \end{pmatrix},$$

then the tile-sum can be explicitly evaluated and shown to be equal to sum over skeleton diagrams. Proven to give the dominant contribution in the 'low density' regime, while conjectured at any density of perturbed gates.

Path integral (aka skeleton) formula for correlation functions

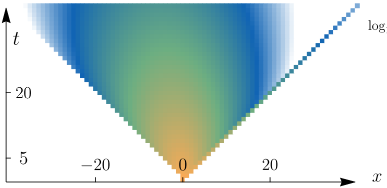
Under the above conditions we *recursively block diagonalize* TMs:

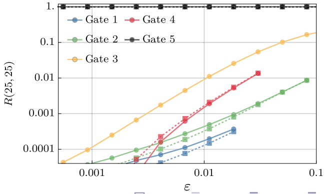


and obtain:

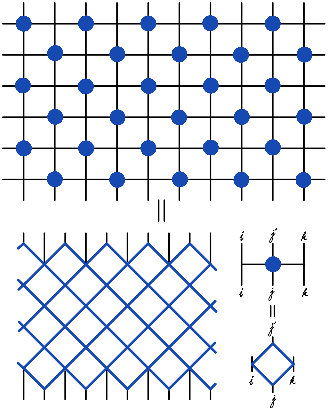
$$\langle \bullet_x | \bullet_0(t)\rangle = \begin{cases} a^{x_+}\delta_{x_-,-1} + \sum_{n=1}^{\bar{n}_1} (pq\varepsilon^2)^n \binom{\tilde{x}_+}{n}\binom{\tilde{x}_- - 2}{n-1} a^{x_+ - n} c^{x_- - n - 1} & x \in \mathbb{Z}, \\ q\varepsilon \sum_{n=0}^{\bar{n}_2} (pq\varepsilon^2)^n \binom{\tilde{x}_+ - 1}{n}\binom{\tilde{x}_- - 1}{n} a^{x_+ - n - 1} c^{x_- - n - 1} & x \in \mathbb{Z} + 1/2, \end{cases}$$

$$\tilde{x}_\pm := \lfloor x_\pm / \nu_\pm \rfloor, \quad x_\pm := t \pm x, \quad \bar{n}_{1,2} \simeq \min(\tilde{x}_+, \tilde{x}_-)$$



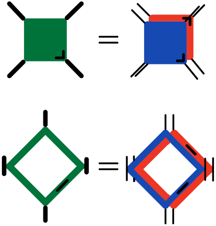


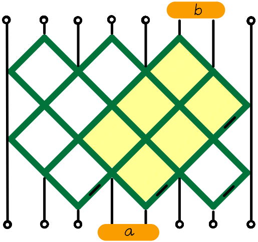
[TP, Chaos 2021]



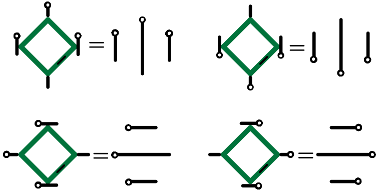
$$U^{\mathrm{IRF}} = \sum_{i,j,k,j'} (u_{ik})^{j'}_{j}\, |i\rangle \otimes |j'\rangle \otimes |k\rangle\langle i| \otimes \langle j| \otimes \langle k|, \quad u_{ik} \in \mathrm{U}(d)$$

An example of IRF circuits: reversible 3-site Margolus cellular automata, cf. Rule 54 - reviewed in [JSTAT (2021) 074001].

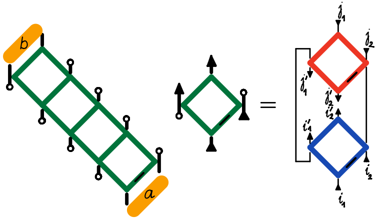



b
a

Unitarity and dual-unitarity of IRF gates:



Consequently, only non-vanishing correlators along 2-leg ladders:



- First exact results on spectral statistics of extended quantum lattice systems, when thermodynamic limit taken first

The main challenges for future work:

- Exact results in finite systems, finite size corrections?
- Statements about eigenstates:
  dual unitary circuits as models where ETH[1] can be proven?

[1] Eigenstate thermalization hypothesis

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Conclusions

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[1] Eigenstate thermalization hypothesis